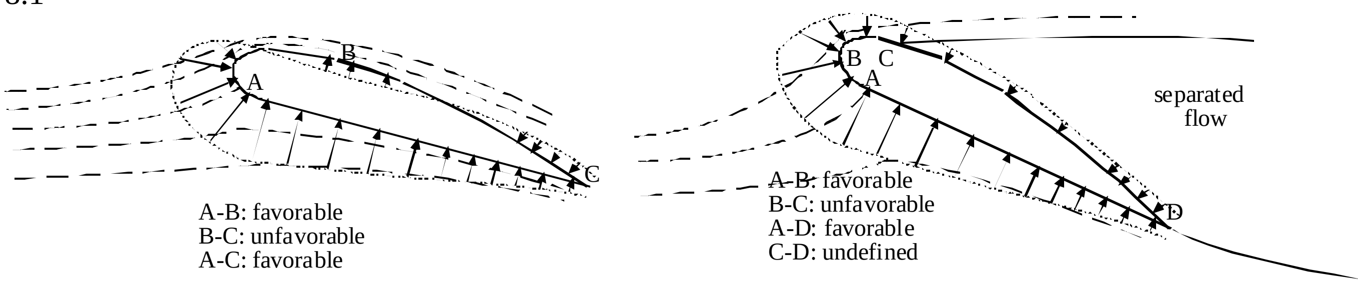
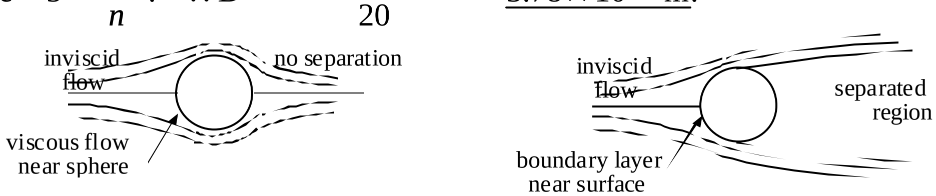
# CHAPTER 8

# External Flows

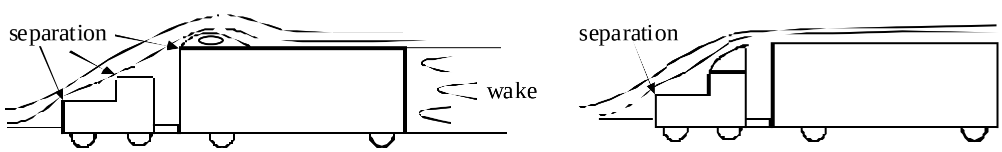
8.1



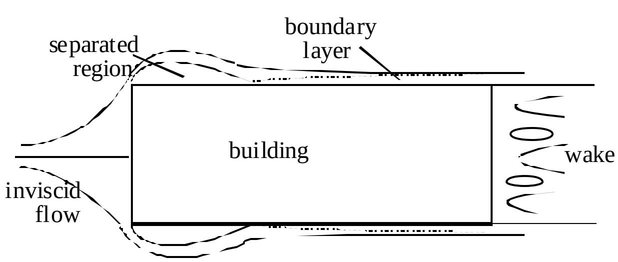

8.2 $\text{Re} = 5 = \dfrac{VD}{n}. \quad \therefore D = \dfrac{5 \times 1.51 \times 10^{-5}}{20} = \underline{3.78 \times 10^{-5}\ \text{m}}.$




8.3




8.4




8.5 (**C**)

8.6 (**C**)

8.7 (**B**) $\text{Re} = \dfrac{VD}{n} = \dfrac{0.8 \times 0.008}{1.31 \times 10^{-6}} = 4880.$

8.8 $5 = \dfrac{VD}{n}$ $\therefore V = 5n \ / \ D.$ a) $V = 5 \times 1.22 \times 10^{-5} \ / \ \dfrac{.8}{12} = \underline{.000915 \text{ fps.}}$

b) $V = 5 \times .388 \times 10^{-5} \ / \ \dfrac{.8}{12} = \underline{.000291 \text{ fps.}}$ c) $V = 5 \times 1.6 \times 10^{-4} \ / \ \dfrac{.8}{12} = \underline{0.012 \text{ fps.}}$

8.9 $\text{Re} = \dfrac{VD}{n} = \dfrac{20 \times D}{1.51 \times 10^{-5}} = 13.25 \times 10^5 D.$

a) $\text{Re} = 13.25 \times 10^5 \times 6 = \underline{7.9 \times 10^6}.$ $\therefore$Separated flow.

b) $\text{Re} = 13.25 \times 10^5 \times .06 = \underline{7.9 \times 10^4}.$ $\therefore$Separated flow.

c) $\text{Re} = 13.25 \times 10^5 \times .006 = \underline{7950}.$ $\therefore$Separated flow.

8.10 $F_D = \displaystyle\int_{A_{\text{front}}} p dA - p_{\text{back}} A_{\text{back}} = p_0 \int_0^1 (1 - r^2) 2prdr = p_0 2p \left( \frac{1}{2} - \frac{1}{4} \right) = \frac{1}{2} pp_0$

Bernoulli: $p_\infty + \dfrac{1}{2} r V_\infty^2 = p_0.$ $\therefore p_0 = \dfrac{1}{2} \times 1.21 \times 20^2 = 242 \text{ Pa.}$

$\therefore F_D = \dfrac{1}{2} p (242) = \underline{380 \text{ N}}$

$C_D = \dfrac{F_D}{\frac{1}{2} r V^2 A} = \dfrac{2 \times 380}{1.21 \times 20^2 \times p \times 1^2} = \underline{0.5}$

8.11 $F_{\text{total}} = F_{\text{bottom}} + F_{\text{top}} = 20\,000 \times .3 \times .3 + 10\,000 \times .3 \times .3 = 2700 \text{ N.}$

$F_{\text{lift}} = 2700 \cos 10^\circ = \underline{2659 \text{ N}}$

$F_{\text{drag}} = 2700 \sin 10^\circ = \underline{469 \text{ N}}$

$C_L = \dfrac{F_L}{\frac{1}{2} r V^2 A} = \dfrac{2 \times 2659}{1000 \times 5^2 \times .3 \times .3} = \underline{2.36}$

$C_D = \dfrac{F_D}{\frac{1}{2} r V^2 A} = \dfrac{2 \times 469}{1000 \times 5^2 \times .3 \times .3} = \underline{0.417}$

8.12 $F_\ell = p_\ell A_\ell = 26\,000 \times Lw.$ $F_u = p_u A_u = 8000 \times \dfrac{Lw}{2 \cos 5^\circ} = 4015 \ Lw$

$F_L = F_\ell \cos 5^\circ - F_u \cos 10^\circ = 21\,950 \ Lw$

$F_D = F_\ell \sin 5^\circ - F_u \sin 10^\circ = 1569 \ Lw$

$$C_L = \frac{F_L}{\frac{1}{2} r V^2 A} = \frac{2 \times 21\ 950\ Lw}{.3119 \times 750^2 Lw} = \underline{0.25}$$

$$C_D = \frac{F_D}{\frac{1}{2} r V^2 A} = \frac{2 \times 1569 Lw}{.3119 \times 750^2 Lw} = \underline{0.0179}$$

8.13 If $C_D = 1.0$ for a sphere, Re = 100 (see Fig. 8.8). $\therefore \frac{V \times .1}{n} = 100,\ V = 1000n$.

a) $V = 1000 \times 1.46 \times 10^{-5} = .0146\ \text{m / s}. \quad \therefore F_D = \frac{1}{2} \times 1.22 \times .0146^2 p \times .05^2 \times 1.0$

$$= \underline{3.25 \times 10^{-7}\ \text{N}}.$$

b) $V = 1000 \times \frac{1.46 \times 10^{-5}}{.015 \times 1.22} = 0.798\ \text{m / s}. \quad \therefore F_D = \frac{1}{2} \times (.015 \times 1.22) \times .798^2 p \times .05^2 \times 1.0$

$$= \underline{4.58 \times 10^{-5}\ \text{N}}.$$

c) $V = 1000 \times 1.31 \times 10^{-6} = .00131\ \text{m / s}. \quad \therefore F_D = \frac{1}{2} \times 1000 \times .00131^2 p \times .05^2 \times 1.0$

$$= \underline{6.74 \times 10^{-6}\ \text{N}}.$$

8.14 a) $\text{Re} = \frac{VD}{n} = \frac{6 \times .5}{1.5 \times 10^{-5}} = 2 \times 10^5. \quad \therefore C_D = 0.45$ from Fig. 8.8.

$$\therefore F_D = \frac{1}{2} r\ V^2\ AC_D = \frac{1}{2} \times 1.22 \times 6^2 \times p \times .25^2 \times .45 = 1.94\ \text{N}.$$

b) $\text{Re} = \frac{15 \times .5}{1.5 \times 10^{-5}} = 5 \times 10^5. \quad \therefore C_D = 0.2$ from Fig. 8.8.

$$\therefore F_D = \frac{1}{2} r\ V^2\ AC_D = \frac{1}{2} \times 1.22 \times 15^2 \times p \times .25^2 \times .2 = 5.4\ \text{N}.$$

8.15 (**B**) Assume a large Reynolds number so that $C_D = 0.2$. Then

$$F = \frac{1}{2} rV^2 AC_D = \frac{1}{2} \times 1.23 \times \left(\frac{80 \times 1000}{3600}\right)^2 \times p \times 5^2 \times 0.2 = 4770\ \text{N}.$$

8.16 (**D**) Assume a Reynolds number of $10^5$. Then $C_D = 1.2$.

$$F = \frac{1}{2} rV^2 AC_D. \quad \therefore 60 = \frac{1}{2} \times 1.23 \times 40^2 \times 4 \times D \times 1.2. \quad \therefore D = 0.0041\ \text{m}.$$

$$\text{Re} = \frac{VD}{n} = \frac{40 \times 0.0041}{10^{-6}} = 1.64 \times 10^5. \quad \therefore C_D = 1.2. \quad \text{The assumption was OK.}$$

8.17 The velocities associated with the two Re's are

$$V_1 = \frac{\mathrm{Re}_1 \nu}{D} = \frac{3\times 10^5 \times 1.5\times 10^{-5}}{.0445} = 101 \text{ m / s},$$

$$V_2 = \frac{\mathrm{Re}_2 \nu}{D} = \frac{6\times 10^4 \times 1.5\times 10^{-5}}{.0445} = 20 \text{ m / s}.$$

The drag, between these two velocities, is reduced by a factor of 2.5 $\left[(C_D)_{\text{high}} = 0.5 \text{ and } (C_D)_{\text{low}} = 0.2\right]$. Thus, between 20 m/s and 100 m/s the drag is reduced by a factor of 2.5. This would significantly lengthen the flight of the ball.

8.18 a) $F_D = \frac{1}{2}\rho V^2 AC_D$. $\therefore 0.5 = \frac{1}{2}\times .00238V^2\pi \times \left(\frac{2}{12}\right)^2 C_D$. $\therefore V^2 C_D = 4810.$

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{V\times 4\ /\ 12}{1.6\times 10^{-4}} = 2080V. \quad \text{Try } C_D = .5\text{:} \quad V = 98 \text{ fps, Re} = 2\times 10^5.$$

$$\text{Try } C_D = .4\text{:} \quad V = \underline{110 \text{ fps}}, \text{ Re} = 2.3\times 10^5.$$

b) $C_D = 0.2$: $0.5 = \frac{1}{2}\times .00238V^2\pi \times \left(\frac{2}{12}\right)^2 \times .2.$ $\therefore V = \underline{155 \text{ fps.}}$

8.19 $4.2 = \frac{1}{2}\times 1000V^2\pi\times .1^2 C_D$. $\therefore V^2 C_D = 0.267.$ $\mathrm{Re} = \frac{V\times .2}{10^{-6}} = 2\times 10^5 V.$

Try $\underline{C_D = 0.5}$: $\therefore V = 0.73$ m / s. $\mathrm{Re} = \underline{1.46\times 10^5}$. $\therefore$ OK.

8.20 $\mathrm{Re} = \frac{VD}{\nu} = \frac{40\times 2}{1.5\times 10^{-5}} = 5.3\times 10^6$. $\therefore C_D = 0.7$. (This is extrapolated from Fig. 8.8.) $\therefore F_D = \frac{1}{2}\times 1.22\times 40^2 \times (2\times 60)\times .7 = \underline{81\ 900 \text{ N}}.$

$$M = 81\ 900 \times 30 = \underline{2.46\times 10^6 \text{ N}\cdot\text{m}}.$$

8.21 a) $\mathrm{Re}_1 = \frac{25\times .05}{1.08\times 10^{-5}} = 1.2\times 10^5$. $\mathrm{Re}_2 = 1.8\times 10^5$. $\mathrm{Re}_3 = 2.4\times 10^5$. Assume a rough cylinder (the air is highly turbulent).

$$\therefore (C_D)_1 = 0.7,\ (C_D)_2 = 0.8,\ (C_D)_3 = 0.9.$$

$$\therefore F_D = \frac{1}{2}\times 1.45\times 25^2(.05\times 10\times .7 + .075\times 15\times .8 + .1\times 20\times .9) = \underline{1380 \text{ N}}.$$

$$M = \frac{1}{2}\times 1.45\times 25^2(.05\times 10\times .7\times 40 + .075\times 15\times .8\times 27.5 + .1\times 20\times .9\times 10) = \underline{25\ 700 \text{ N}\cdot\text{m}}.$$

b) $\mathrm{Re}_1 = \frac{25\times .05}{1.65\times 10^{-5}} = 7.6\times 10^4$. $\mathrm{Re}_2 = 1.14\times 10^5$, $\mathrm{Re}_3 = 1.5\times 10^5$.

$\therefore (C_D)_1 = .8,\ (C_D)_2 = .7,\ (C_D)_3 = .8. \quad \rho = \frac{101}{.287 \times 308} = 1.17 \text{ kg / m}^3.$

$\therefore F_D = \frac{1}{2} \times 1.17 \times 25^2 (.05 \times 10 \times .8 + .075 \times 15 \times .7 + .1 \times 20 \times .8) = \underline{1020 \text{ N}}.$

$M = \frac{1}{2} \times 1.17 \times 25^2 (.05 \times 10 \times .8 \times 40 + .075 \times 15 \times .7 \times 27.5 + .1 \times 20 \times .8 \times 10) = \underline{19\ 600 \text{ N} \cdot \text{m}}.$

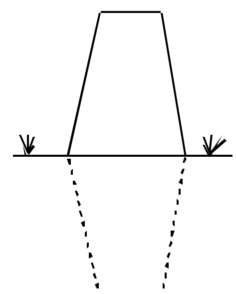
8.22 Atmospheric air is turbulent. $\therefore$ Use the "rough" curve. $\therefore C_D = 0.7.$

$F_D = 10 = \frac{1}{2} \times .00238 V^2 \times 6D \times .7. \quad \therefore 2000 = V^2 D. \quad 10^5 = \frac{V \times 2000 / V^2}{1.6 \times 10^{-4}}.$

$\therefore p_{\min} = \frac{\rho}{2}\left[U_\infty^2 - v_0^2\right] = \frac{0.0024}{2}\left[30^2 - 104^2\right] = \underline{-11.8 \text{ psf}}.$

$\therefore V^2 D = 2370. \quad \therefore V = \underline{148 \text{ fps}}. \quad D = \underline{0.108'}.$

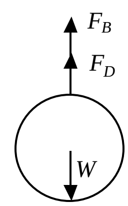
8.23 Since the air cannot flow around the bottom, we imagine the structure to be mirrored as shown. Then $L / D = 40 / 5 = 8. \quad \therefore C_D = 0.66 C_{D_\infty}.$



$\text{Re}_{\min} = \frac{V D_{\min}}{\nu} = \frac{30 \times 2}{1.5 \times 10^{-5}} = 4 \times 10^6. \quad \therefore C_D = 1.0 \times .66 = .66.$

$\therefore F_D = \frac{1}{2} \times 1.22 \times 30^2 \times \left(\frac{2+8}{2} \times 20\right) \times .66 = \underline{36\ 000 \text{ N}}.$

8.24 $F_B + F_D = F_W.$




$9810 \times \frac{4}{3}\pi r^3 + \frac{1}{2} \times 1000 V^2 \pi r^2 C_D = 9810 \times 7.82 \times \frac{4}{3}\pi r^3.$

$\text{Re} = \frac{V \times 2r}{10^{-6}} = 2 \times 10^6 Vr. \quad \therefore V^2 C_D = 178 r$

a) $r = .05$ m. $\therefore \text{Re} = 10^5 V,\ V^2 C_D = 8.9.$ Assume a smooth sphere.

Try $C_D = .5$: $\therefore V = 4.22$ m/s. $\text{Re} = 4.22 \times 10^5$. This is too large for Re.

Try $C_D = .2$: $\therefore V = \underline{6.67 \text{ m/s}}$. $\text{Re} = 6.67 \times 10^5$. OK.

b) $r = .025$ m. $\text{Re} = 5 \times 10^4 V,\ V^2 C_D = 4.45.$

Try $C_D = .2$: $V = \underline{4.72 \text{ m/s}}$. $\text{Re} = 2.4 \times 10^5$. OK.

c) $r = .005$ m. $\text{Re} = 10^4 V,\ V^2 C_D = 0.89.$

Try $C_D = .5$: $V = \underline{1.33 \text{ m/s}}$. $\text{Re} = 1.33 \times 10^4$. OK.

d) $r = .001$ m. $\text{Re} = 2 \times 10^3 V,\ V^2 C_D = 0.178.$

Try $C_D = .4$: $V = \underline{0.67 \text{ m/s}}$. $\text{Re} = 1.33 \times 10^3$. OK.

8.25 $F_B + F_D = F_W$. $.077\times\frac{4}{3}\pi\left(\frac{10}{12}\right)^3 + \frac{1}{2}\times.00238V^2\pi\left(\frac{10}{12}\right)^2 C_D = 62.4S\frac{4}{3}\pi\left(\frac{10}{12}\right)^3$.

$$\text{Re} = \frac{V\times10/12}{1.6\times10^{-4}} = 5.2\times10^3V. \qquad \therefore 1+.0139V^2C_D = 810S$$

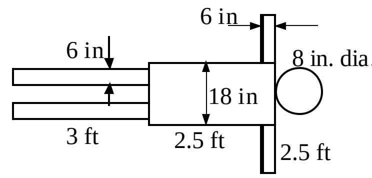
a) $S = .005$. $V^2C_D = 219$. Assume atmospheric turbulence, i.e., rough.
Try $C_D = .4$: $V = 23.4$ fps. $\text{Re} = 1.2\times10^5$. $\therefore C_D = .3$. $V = \underline{27\text{ fps}}$.

b) $S = .02$. $V^2C_D = 1090$. Try $C_D = .4$: $V = \underline{52\text{ fps}}$. $\text{Re} = 2.7\times10^5$. $\therefore$ OK.

c) $S = 1.0$. $V^2C_D = 58\ 200$. Try $C_D = .4$: $V = \underline{381\text{ fps}}$.

8.26

Assume a 180 lb, 6' sky diver, with components as shown. If $V$ is quite large, then Re > $2\times10^5$.
$F_D = F_W$.




$$\frac{1}{2}\times.00238V^2\left[2\times3\times\frac{1}{2}\times1.0\times.7 + 2\times2.5\times\frac{1}{2}\times1.0\times.7 + \frac{18}{12}\times2.5\times1.0 + \pi\times\left(\frac{4}{12}\right)\times.4\right] = 180.$$

We used data from Table 8.1. $\therefore V = \underline{140\text{ fps}}$.

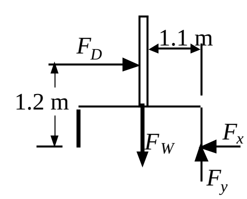
8.27 From Table 8.2 $C_D = 0.35$. $F_D = \frac{1}{2}\times1.22V^2\times3.2\times0.35 = .683V^2$.

a) $F_D = .683\times\left(\frac{80\times1000}{3600}\right)^2 = 337$ N. $\therefore \dot{W} = 337\frac{80\times1000}{3600} = 7500$ W or $\underline{10\text{ Hp}}$.

b) $V = 25$ m / s. $F_D = .683\times25^2 = 427$ N. $\therefore \dot{W} = 427\times25 = 10\ 700$ W or $\underline{14.3\text{ Hp}}$.

c) $V = 27.8$ m / s. $F_D = .683\times27.8^2 = 527$ N. $\therefore \dot{W} = 527\times27.8 = 14\ 700$ W or $\underline{19.6\text{ Hp}}$.

8.28 $1.2F_D = 1.1\times400$. $F_D = \frac{1}{2}\rho V^2AC_D$. $C_D = 1.1$




$1.2\times\frac{1}{2}\times1.22V^2\times(2\times3)\times1.1 = 1.1\times400.$

$\therefore V = \underline{9.5\text{ m/s}}$.

8.29 $\text{Re} = \frac{VD}{\nu} = \frac{(40\ 000\ /\ 3600)0.6}{1.51\times10^{-5}} = 4.42\times10^5$. $\therefore C_D = 0.35$ from Fig. 8.8.

a) $F_D = \frac{1}{2}\rho V^2AC_D = \frac{1}{2}\times1.204\times(40\ 000\ /\ 3600)^2\times0.6\times6\times0.35 = \underline{93.6\text{ N}}$

b) $F_D = 93.6\times0.68 = \underline{63.7\text{ N}}$ where $L\ /\ D = 6\ /\ 0.6 = 10$.

c) $F_D = 93.6\times0.76 = \underline{71.1\text{ N}}$ where we can use $L\ /\ D = 20$ since only one end is free. The ground acts like the mid-section of a 12-m-long cylinder.

8.30 a) Curled up, she makes an approximate sphere of about 1.2 m in diameter (just a guess!). Assume a rough sphere at large Re. From Fig. 8.8, $C_D = 0.4$:

$$F_D = \frac{1}{2} r V^2 A C_D$$

$$80 \times 9.8 = \frac{1}{2} \times 1.21 \times V^2 p \times 0.6^2 \times 0.4. \qquad \therefore V = \underline{53.7 \text{ m / s}}.$$

Check Re: $\text{Re} = \dfrac{53.7 \times 1.2}{1.51 \times 10^{-5}} = 4.27 \times 10^6. \qquad \therefore$ OK.

b) $F_D = \dfrac{1}{2} r V^2 A C_D$. From Table 8.2, $C_D$ = 1.4:

$$80 \times 9.8 = \frac{1}{2} \times 1.21 \times V^2 p \times 4^2 \times 1.4. \qquad \therefore V = \underline{4.29 \text{ m / s}}.$$

Check Re: $\text{Re} = \dfrac{4.29 \times 8}{1.51 \times 10^{-5}} = 2.27 \times 10^6.$ Should be larger but the velocity should be close.

c) $F_D = \dfrac{1}{2} r V^2 A C_D$

$$80 \times 9.8 = \frac{1}{2} \times 1.21 \times V^2 p \times 1^2 \times 1.4. \qquad \therefore V = \underline{17.2 \text{ m/s}}.$$

Check Re: $\text{Re} = \dfrac{17.2 \times 1}{1.51 \times 10^{-5}} = 1.14 \times 10^6.$ This should be greater than $10^7$ for $C_D$ to be acceptable. Hence, the velocity is approximate.

8.31 With the deflector the drag coefficient is 0.76 rather than 0.96. The required power (directly related to fuel consumed) is reduced by the ratio of 0.76 / 0.96. The cost per year without the deflector is

Cost = (200 000 / 1.2) × 0.25 = \$41,667.

With the deflector it is

Cost = 41,667 × 0.76 / 0.96 = \$32,986.

The savings is \$41.667 – 32,986 = \$8,800.

8.32 $F_D = \dfrac{1}{2} r V^2 A C_D = \dfrac{1}{2} \times .00238 \times 88^2 \times (6 \times 2) \times 1.1 = 122 \text{ lb}.$

$\dot{W} = F_D \times V = 122 \times 88 = 10{,}700 \ \dfrac{\text{ft - lb}}{\text{sec}}$ or $\underline{19.5 \text{ Hp}}$.

8.33 $F_D = \dfrac{1}{2} r V^2 A C_D = \dfrac{1}{2} \times 1.22 \times (27.8 \times 1.6)^2 \times p \times .05^2 \times 1.1 = 10.43 \text{ N}.$

$\dot{W} = F_D \times V \times 2 = 10.43 \times (27.8 \times 1.6) \times 2 = 226 \text{ W}$ or $\underline{1.24 \text{ Hp}}$.

8.34 The projected area is $\frac{(2+0.3)}{2}\times 4 = 4.6\ \text{m}^2$.

$$F_D = \frac{1}{2}\rho V^2 A C_D = \frac{1}{2}\times 1.18\times 20^2 \times 4.6\times 0.4 = 434\ \text{N}.$$

Since there are two free ends, we use Table 8.1 with $L/D = 4/1.15 = 3.47$, and approximate the force as

$$F_D = 434\times 0.62 = \underline{269\ \text{N}}.$$

8.35 The net force acting up is (use absolute pressure)

$$F_{\text{up}} = \frac{4}{3}\pi\times 0.4^3\times 1.21\times 9.8 - 0.5 - \frac{4}{3}\pi\times 0.4^3\,\frac{120}{2.077\times 293}\times 9.8 = 2.16\ \text{N}$$

From a force triangle (2.16 N up and $F_D$ to the right), we see that

$$\tan\alpha = F_{\text{up}}/F_D.$$

a) $F_D = 2.16/\tan 80^\circ = 0.381.$

$$0.381 = \frac{1}{2}\times 1.21 V^2\pi\times 0.4^2\times 0.2. \qquad \therefore V = 2.50\ \text{m/s}.$$

Check Re: $\text{Re} = \frac{2.5\times 0.8}{1.51\times 10^{-5}} = 1.33\times 10^5$. Too low. Use $C_D = 0.5$:

$$0.381 = \frac{1}{2}\times 1.21 V^2\pi\times 0.4^2\times 0.5. \qquad \therefore V = \underline{1.58\ \text{m/s}}$$

b) $F_D = 2.16/\tan 70^\circ = 0.786.$

$$0.786 = \frac{1}{2}\times 1.21 V^2\pi\times 0.4^2\times 0.2. \qquad \therefore V = 3.60\ \text{m/s}.$$

Check Re: $\text{Re} = \frac{3.6\times 0.8}{1.51\times 10^{-5}} = 1.9\times 10^5$. Too low. Use $C_D = 0.5$:

$$0.786 = \frac{1}{2}\times 1.21 V^2\pi\times 0.4^2\times 0.5. \qquad \therefore V = \underline{2.27\ \text{m/s}}$$

c) $F_D = 2.16/\tan 60^\circ = 1.25.$

$$1.25 = \frac{1}{2}\times 1.21 V^2\pi\times 0.4^2\times 0.5. \qquad \therefore V = \underline{2.86\ \text{m/s}}.$$

Check Re: $\text{Re} = \frac{2.86\times 0.8}{1.51\times 10^{-5}} = 1.5\times 10^5$. $\therefore$OK.

d) $F_D = 2.16/\tan 50^\circ = 1.81.$

$$1.81 = \frac{1}{2}\times 1.21 V^2\pi\times 0.4^2\times 0.5. \qquad \therefore V = \underline{3.45\ \text{m/s}}.$$

Check Re: $\text{Re} = \frac{3.45\times 0.8}{1.51\times 10^{-5}} = 1.8\times 10^5$. Close, but OK.

8.36 Assume each section of the tree is a cylinder. The average diameter of the tree is 1 m. The top doesn't have a blunt end around which the air flows, however, the bottom does; so assume $L/D = (5/2)\times 2 = 5$. So, use a factor of 0.62 from Table 8.1 to multiply the drag coefficient. The force acts near the centroid of the triangular area, one-third the way up. Finally,

$$F \times d = 5000$$

$$\left[\frac{1}{2}\times 1.21V^2(5)\times 0.4\times 0.62\right]\times\left(\frac{5}{3}+0.6\right) = 5000. \quad V = \underline{54.2 \text{ m / s}}.$$

8.37 Power to move the sign:

$$F_D V = \frac{1}{2}\rho V^2 A C_D \times V$$

$$= \frac{1}{2}\times 1.21\times 11.11^2 \times 0.72\times 1.1\times 11.11 = 657 \text{ J / s}.$$

This power comes from the engine:

$$657 = (12\,000\times 1000)\dot{m}\times 0.3. \qquad \therefore \dot{m} = 1.825\times 10^{-4} \text{ kg / s}.$$

Assuming the density of gas to be 900 $\text{kg}/\text{m}^3$,

$$1.825\times 10^{-4}\times 10\times 3600\times 6\times 52\times \frac{1000}{900}\times 0.30 = \underline{\$683}$$

8.38 The power expended is $F_D \times V$. $V = (25\times 88/60)/3.281 = 11.18$ m / s

$$\frac{1}{2}\times 1.21\times 11.18^3\times 0.56\times C_D = \frac{1}{2}\times 1.21\times V^3\times 0.4\times C_D\times 0.8$$

$$\therefore V = 13.47 \text{ m/s or } \underline{30.1 \text{ mph}}.$$

8.39 $\dot{W} = 40\times 746\eta = F_D \times V = \frac{1}{2}\rho V^2 A C_D \times V = \frac{1}{2}\rho A C_D V^3.$

$\therefore 40\times 746\times .9 = \frac{1}{2}\times 1.22\times 3\times 0.35V^3. \quad \therefore V = 34.7 \text{ m/s or } \underline{125 \text{ km/hr}}.$

8.40 (**C**) $\text{Re} = \frac{VD}{\nu} = \frac{4\times 0.02}{1.6\times 10^{-5}} = 5000. \quad \therefore \text{St} = 0.21 = \frac{fD}{V} = \frac{f\times 0.02}{4}.$

$\therefore f = 42$ Hz (cycles/second). distance $= \frac{V}{f} = \frac{4 \text{ m/s}}{42 \text{ cycles/s}} = 0.095$ m/cycle.

8.41 $40 < \text{Re} < 10\,000. \quad 40 < \frac{V\times .003}{1.5\times 10^{-5}} < 10\,000. \quad \therefore \underline{0.2 < V < 50 \text{ m/s}}.$

$\text{St} = 0.12 = \frac{f\times .003}{.2}. \quad \therefore f_{\text{low}} = 8 \text{ Hz}.$

$\text{St} = .21 = \frac{f\times .003}{50}. \quad \therefore f_{\text{lhigh}} = 3500 \text{ Hz}.$

The vortices could be heard over most of the range.

8.42 $40 > \frac{VD}{\nu} = \frac{6D}{1.22\times10^{-5}}$. $\therefore \underline{D < 8.13\times10^{-5} \text{ ft}}$.

$10\ 000 < \frac{VD}{\nu} = \frac{6D}{1.22\times10^{-5}}$. $\therefore D > 0.020$ ft or $\underline{0.24"}$.

8.43 From Fig. 8.9, Re is related to St. $\text{St} = \frac{f\times D}{V} = \frac{0.2\times.1}{V}$.

$\text{Re} = \frac{VD}{\nu} = \frac{V\times.1}{1.5\times10^{-5}}$. Try St =.21: $V = 0.095$ m / s. $\therefore$ Re = 630.

This is acceptable. $\therefore V = \underline{0.095 \text{ m / s}}$.

8.44 $\text{St} = \frac{fD}{V} = \frac{.002\times2}{V}$. $\text{Re} = \frac{VD}{\nu} = \frac{V\times2}{10^{-6}}$. Use Fig. 8.9.

Try St = 0.21: $V = \underline{.0191 \text{ m / s}}$. $\text{Re} = 38\times10^3$. $\therefore$ OK.

8.45 Let $S_t = 0.21$ for the wind imposed vorticies. When this frequency equals the natural frequency, or one of its odd harmonics, resonance occurs:

$$f = \sqrt{T / \rho L^2 d^2 \pi}$$

$$\frac{0.21\times10}{0.016} = \sqrt{30\ 000/7850L^2\times0.016^2\times\pi}. \quad \therefore L = \underline{0.525 \text{ m}}$$

Consider the third and fifth harmonics:

$$f = 3\sqrt{T / \rho L^2 d^2 \pi}. \quad \therefore L = \underline{1.56 \text{ m}}.$$

$$f = 5\sqrt{T / \rho L^2 d^2 \pi}. \quad \therefore L = \underline{2.62 \text{ m}}.$$

8.46 (**C**) By reducing the separated flow area, the pressure in that area increases thereby reducing that part of the drag due to pressure.

Fig. 8.8 Table 8.1

8.47 $\text{Re} = \frac{88\times6\ /\ 12}{1.6\times10^{-4}} = 2.8\times10^5$. $F_D = \frac{1}{2}\times.00238\times88^2\times1.0\times.8\times\left(6\times\frac{6}{12}\right) = 22$ lb.

The coefficient 1.0 comes from Fig. 8.8 and 0.8 from Table 8.1.

$\dot{W} = F_D\times V = 22\times88 = 1946$ ft - lb / sec or $\underline{3.5 \text{ Hp}}$.

$(C_D)_{\text{streamlined}} = 0.035$. $\therefore F_D = 0.77$ lb. $\dot{W} = 67.8 \frac{\text{ft - lb}}{\text{sec}}$ or $\underline{0.12 \text{ Hp}}$.

8.48 $\text{Re} = \frac{VD}{\nu} = \frac{3\times.08}{1.5\times10^{-5}} = 16\ 000$. $\therefore F_D = \frac{1}{2}\times1.22\times3^2\times(0.08\times2)\times1.2\times.78 = \underline{0.822 \text{ N}}$

The coefficient 1.2 comes from Fig. 8.8 and 0.78 from Table 8.1.

$(C_D)_{\text{streamlined}} = .35$. $\therefore F_D = 0.24$ N. $\therefore$ % reduction $= \frac{0.822-0.24}{0.822}\times100 = \underline{70.8\%}$

8.49 $\text{Re} = \frac{VD}{n} = \frac{2\times 0.8}{10^{-6}} = 1.6\times 10^{6}.$ $\therefore C_D = 0.45$ from Fig. 8.8.

$\frac{L}{D} = \frac{4}{0.8} = 5.$ $\therefore C_D = 0.62\times 0.45 = 0.28.$

Because only one end is free, we double the length.

$F_D = \frac{1}{2} rV^2 AC_D = \frac{1}{2}\times 1000\times 2^2\times 0.8\times 2\times 0.28 = \underline{900\text{ N}}.$

If streamlined, $C_D = 0.03\times 0.62 = 0.0186.$

$\therefore F_D = \frac{1}{2}\times 1000\times 2^2\times 0.8\times 2\times 0.0186 = \underline{60\text{ N}}.$

8.50 $V = 50\times 1000\ /\ 3600 = 13.9\text{ m / s}.$

Assume the ends to not be free. $\therefore$ Use $C_D$ from Fig. 8.8.

$\text{Re} = \frac{13.9\times 0.02}{1.5\times 10^{-5}} = 1.85\times 10^4.$ $\therefore C_D = 1.2.$ $(C_D)_{\text{streamlined}} = 0.3$

$\dot{W} = F_D\times V = \frac{1}{2} rV^3 AC_D = \frac{1}{2}\times 1.2\times 13.9^3\times 0.02\times 20\times 1.2 = 773\text{ W}$ or $\underline{1.04\text{ Hp}}.$

$\dot{W}_{\text{streamlined}} = \frac{1}{2}\times 1.2\times 13.9^3\times 0.02\times 20\times 0.3 = 193\text{ W}$ or $\underline{0.26\text{ Hp}}$

8.51 $V = 50\times 1000 / 3600 = 13.9\text{ m / s}.$ $\text{Re} = \frac{13.9\times 0.3}{1.5\times 10^{-5}} = 2.8\times 10^5.$ $\therefore C_D = 0.4$

We assumed a head diameter of 0.3 m and used the rough sphere curve.

$F_D = \frac{1}{2}\times 1.2\times 13.9^2 (p\times 0.3^2 / 4)\times 0.4 = \underline{3.3\text{ N}}.$

$F_D = \frac{1}{2}\times 1.2\times 13.9^2 (p\times 0.3^2 / 4)\times 0.035 = \underline{0.29\text{ N}}.$

8.52 $s = \frac{p_\infty - p_v}{\frac{1}{2} rV^2}.$ $0.7 = \frac{150\ 000 - 1670}{\frac{1}{2}\times 1000V^2}$ where $p_\infty = g\,h + p_{\text{atm}} = 150\ 000\text{ Pa}.$

$\therefore V = \underline{20.6\text{ m / s}}.$

8.53 $C_L = \frac{F_L}{\frac{1}{2} rV^2 A} = \frac{200\ 000}{\frac{1}{2}\times 1000\times 12^2\times .4\times 10} = 0.69.$ $\therefore \underline{a \cong 3^\circ}.$

$C_D = .0165 = \frac{F_D}{\frac{1}{2}\times 1000\times 12^2\times .4\times 10}.$ $\therefore F_D = \underline{4800\text{ N}}.$

$$s_{\text{crit}} = .75 \overset{?}{>} \frac{(9810 \times .4 + 101\,000) - 1670}{\frac{1}{2} \times 1000 \times 12^2} = 1.43. \qquad \therefore \underline{\text{No cavitation.}}$$

8.54 $$C_L = \frac{F_L}{\frac{1}{2} r V^2 A} = \frac{50\,000}{\frac{1}{2} \times 1.94 \times 35^2 \times \frac{16}{12} \times 30} = 1.05. \qquad \therefore a = 7.3^\circ.$$

$$C_D = .027 = \frac{F_D}{\frac{1}{2} \times 1.94 \times 35^2 \times \frac{16}{12} \times 30}. \qquad \therefore F_D = \underline{1280 \text{ lb}}.$$

$$s_{\text{crit}} = 1.6 \overset{?}{>} \frac{62.4 \times 16/12 + 2117 - .25 \times 144}{\frac{1}{2} \times 1.94 \times 35^2} = 1.82. \qquad \therefore \underline{\text{No cavitation.}}$$

8.55 $$p_\infty = 9810 \times 5 + 101\,000 = 150\,000 \text{ Pa}. \quad p_v = 1670 \text{ Pa}. \quad \text{Re} = \frac{20 \times .8}{10^{-6}} = 16 \times 10^6.$$

$$s = \frac{150\,000 - 1670}{\frac{1}{2} \times 1000 \times 20^2} = 0.74. \qquad \therefore C_D = C_D(0)(1 + s) = .3(1 + .74) = .52$$

$$\therefore F_D = \frac{1}{2} r V^2 A C_D = \frac{1}{2} \times 1000 \times 20^2 \times p \times .4^2 \times .52 = \underline{52\,000 \text{ N}}.$$

Note: We retain 2 sig. figures since $C_D$ is known to only 2 sig. nos.

8.56 For a 6° angle of attack we find from Table 8.4 $C_L = 0.95$.

$$F_L = \frac{1}{2} r V^2 A C_L = \frac{1}{2} \times 1000 \times 15^2 \times 4 \times 0.4 L \times .95 = 12\,000 \times 9.8.$$

$$\therefore L = \underline{0.69 \text{ m}}.$$

8.57 $$\Sigma F = ma. \quad \text{a) } 400 - 9810 \times \frac{4}{3} p \times .2^3 = \frac{400}{9.81} a. \qquad \therefore a = \underline{1.75 \text{ m/s}^2}.$$

$$\text{b) } 400 - 9810 \times \frac{4}{3} p \times .2^3 = \left( \frac{400}{9.81} + \frac{1}{2} \times 1000 \times \frac{4}{3} p \times .2^3 \right) a. \qquad \therefore a = \underline{1.24 \text{ m/s}^2}.$$

8.58 $$F = ma_1 = 1000 \times 1.2 \times V a_1. \quad \therefore a_1 = \frac{F}{1200 V}. \quad m_a = 0.2 \times 1000 V.$$

$$F = (m + m_a) a_2. \quad \therefore a_2 = \frac{F}{1200 V + 200 V} = \frac{F}{1400 V}. \quad a_2 \text{ is true acceleration.}$$

$$\therefore \% \text{ error} = \left| \frac{a_2 - a_1}{a_2} \right| \times 100 = \left| \frac{\frac{F}{1400 V} - \frac{F}{1200 V}}{\frac{F}{1400 V}} \right| \times 100 = \underline{16.7\%}.$$

8.59 (**B**) From Fig. 8.12a $C_L = 1.1$. $C_L = \dfrac{F_L}{\frac{1}{2}\rho V^2 cL}$.

$$\therefore V^2 = \frac{2W}{\rho cLC_L} = \frac{2\times1200\times9.81}{1.23\times16\times1.1} = 1088. \quad \therefore V = 33.0 \text{ m/s}.$$

8.60 $$C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A} = \frac{1000\times9.81}{\frac{1}{2}\times.412\times80^2\times15} = 0.496. \qquad \therefore \alpha = 3.2^\circ. \quad C_D = .0065.$$

$$\dot{W} = F_D V = \left(\frac{1}{2}\times.412\times80^2\times15\times.0065\right)\times80 = 10\ 300 \text{ W} \quad \text{or} \quad \underline{13.8 \text{ Hp}}.$$

8.61 a) $$C_L = 1.22 = \frac{1500\times9.81+3000}{\frac{1}{2}\times1.22\times V^2\times20}. \qquad \therefore V = \underline{34.5 \text{ m/s}}.$$

b) $$(C_L)_{\max} = 1.72 = \frac{1500\times9.81+3000}{\frac{1}{2}\times.412\times V^2\times20}. \qquad \therefore V = \underline{50 \text{ m/s}}. \qquad \text{(at 10 000 m)}$$

c) $$\dot{W} = F_D V = \left(\frac{1}{2}\times.412\times80^2\times20\times.0065\right)\times80 = 13\ 700 \text{ W} \quad \text{or} \quad \underline{18.4 \text{ Hp}}$$

where we found $C_D$ as follows:

$$(C_L)_{\text{cruise}} = \frac{1500\times9.81+3000}{\frac{1}{2}\times.412\times80^2\times20} = .67. \qquad \therefore C_D = .0065, \text{ from Fig. 8.12.}$$

$$\therefore \text{Power} = \frac{18.4}{0.45} = \underline{40.9 \text{ Hp}}.$$

8.62 $$C_L = 1.22 = \frac{1500\times9.81+3000}{\frac{1}{2}\times1.007\times V^2\times20}. \qquad \therefore V = \underline{38.0 \text{ m/s}}.$$

8.63 $$(C_L)_{\text{cruise}} = \frac{1500\times9.81+3000}{\frac{1}{2}\times1.007\times80^2\times20} = 0.275. \qquad \therefore C_D = \frac{0.275}{48} = 0.0057.$$

$$\therefore \dot{W} = F_D V = \frac{1}{2}\times1.007\times80^3\times20\times0.0057 = 29\ 400 \text{ W} \quad \text{or} \quad 39.4 \text{ Hp}$$

$$\%\ \text{change} = \frac{39.4-18.4}{18.4}\times100 = \underline{114\%\ \text{increase}}$$

The increased power is due to the increase in air density.

8.64 $C_L = 1.22 = \dfrac{1500 \times 9.81 + 9000}{\frac{1}{2} \times 1.22 \times V^2 \times 20}$. $\quad \therefore V = \underline{39.9 \text{ m/s}}$.

8.65 $C_L = 1.72 = \dfrac{250\ 000 \times 9.81}{\frac{1}{2} \times 1.22 \times V^2 \times 60 \times 8}$. $\quad \therefore V = \underline{69.8 \text{ m/s}}$.

8.66 a) $C_L = 1.72 = \dfrac{250\ 000 \times 9.81}{\frac{1}{2} \times 1.05 \times V^2 \times 60 \times 8}$. $\quad \therefore V = 75.2 \text{ m/s}$.

$$\%\text{ change} = \frac{75.2 - 69.8}{69.8} \times 100 = \underline{7.77\% \text{ increase}}$$

b) $C_L = 1.72 = \dfrac{250\ 000 \times 9.81}{\frac{1}{2} \times 1.515 V^2 \times 60 \times 8}$. $\quad \therefore V = 62.6 \text{ m/s} \quad \left( r = \dfrac{101.3}{.287 \times 233} = 1.515 \text{ kg/m}^3 \right)$

$$\%\text{ change} = \frac{62.6 - 69.8}{69.8} \times 100 = \underline{-10.3\%}$$

c) $C_L = 1.72 = \dfrac{250\ 000 \times 9.81}{\frac{1}{2} \times 1.093 V^2 \times 60 \times 8}$. $\quad \therefore V = 73.7 \text{ m/s} \quad \left( r = \dfrac{101.3}{.287 \times 323} = 1.093 \text{ kg/m}^3 \right)$

$$\%\text{ change} = \frac{73.7 - 69.8}{69.8} \times 100 = \underline{5.63\% \text{ increase}}$$

8.67 For a conventional airfoil assume $C_L / C_D = 47.6$ at $C_L = 0.3$.

$$0.3 = \frac{m \times 9.81}{\frac{1}{2} \times 0.526 \times 222^2 \times 200 \times 30}. \quad \therefore m = \underline{2.38 \times 10^6 \text{ kg}}$$

$$\dot{W} = F_D V = \frac{1}{2} \times 0.526 \times 222^3 \times 200 \times 30 \times \frac{0.3}{47.6} = 490\ 000 \text{ W} \text{ or } \underline{657 \text{ Hp}}$$

8.68 $\vec{\nabla} \times \left[ \dfrac{\P \vec{V}}{\P t} + (\vec{V} \cdot \vec{\nabla})\vec{V} + \dfrac{\vec{\nabla} p}{r} - n \nabla^2 \vec{V} \right] = 0.$

$$\vec{\nabla} \times \frac{\P \vec{V}}{\P t} = \frac{\P}{\P t}(\vec{\nabla} \times \vec{V}) = \frac{\P \vec{w}}{\P t}. \quad \vec{\nabla} \times \frac{\vec{\nabla} p}{r} = \frac{1}{r} \vec{\nabla} \times \vec{\nabla} p = 0.$$

$\vec{\nabla} \times (\nabla^2 \vec{V}) = \nabla^2 (\vec{\nabla} \times \vec{V}) = \nabla^2 \vec{w}$ (we have interchanged derivatives)

$$\vec{\nabla} \times \left[ (\vec{V} \cdot \vec{\nabla})\vec{V} \right] = \vec{\nabla} \times \left[ \frac{1}{2} \vec{\nabla} V^2 - \vec{V} \times (\vec{\nabla} \times \vec{V}) \right] = \frac{1}{2} \cancel{(\vec{\nabla} \times \vec{\nabla} V^2)} - \vec{\nabla} \times (\vec{V} \times \vec{w})$$

$$= \vec{V} \cancel{(\vec{\nabla} \cdot \vec{w})} - \vec{w} \cancel{(\vec{\nabla} \cdot \vec{V})} + (\vec{V} \cdot \vec{\nabla})\vec{w} - (\vec{w} \cdot \vec{\nabla})\vec{V}$$

$x = L,\ \ u = U. \qquad \therefore \frac{\partial y}{\partial y} = U. = (\vec{V} \cdot \vec{\nabla})\vec{w} - (\vec{w} \cdot \vec{\nabla})\vec{V}$ where $\vec{\nabla} \cdot \vec{w} = \vec{\nabla} \cdot (\vec{\nabla} \times \vec{V}) = 0.$

There results: $\quad \frac{\partial \vec{w}}{\partial t} + (\vec{V} \cdot \vec{\nabla})\vec{w} - (\vec{w} \cdot \vec{\nabla})\vec{V} - n\nabla^2 \vec{w} = 0.$

This is written as $\quad \frac{D\vec{w}}{Dt} = (\vec{w} \cdot \vec{\nabla})\vec{V} + n\nabla^2 \vec{w}.$

8.69 $x$-comp: $\quad \frac{\partial w_x}{\partial t} + u\frac{\partial w_x}{\partial x} + v\frac{\partial w_x}{\partial y} + w\frac{\partial w_x}{\partial z} = w_x\frac{\partial u}{\partial x} + w_y\frac{\partial u}{\partial y} + w_z\frac{\partial u}{\partial z} + n\nabla^2 w_x$

$y$-comp:

$$\frac{\partial w_y}{\partial t} + u\frac{\partial w_y}{\partial x} + v\frac{\partial w_y}{\partial y} + w\frac{\partial w_y}{\partial z} = w_x\frac{\partial v}{\partial x} + w_y\frac{\partial v}{\partial y} + w_z\frac{\partial v}{\partial z} + n\left(\frac{\partial^2 w_y}{\partial x^2} + \frac{\partial^2 w_y}{\partial y^2} + \frac{\partial^2 w_y}{\partial z^2}\right)$$

$z$-comp:

$$\frac{\partial w_z}{\partial t} + u\frac{\partial w_z}{\partial x} + v\frac{\partial w_z}{\partial y} + w\frac{\partial w_z}{\partial z} = w_x\frac{\partial w}{\partial x} + w_y\frac{\partial w}{\partial y} + w_z\frac{\partial w}{\partial z} + n\left(\frac{\partial^2 w_z}{\partial x^2} + \frac{\partial^2 w_z}{\partial y^2} + \frac{\partial^2 w_z}{\partial z^2}\right)$$

8.70 $w_x = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} = 0. \quad w_y = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = 0. \quad w_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \neq 0.$

$\frac{Dw_z}{DT} = (\vec{w} \cdot \vec{\nabla})w + n\nabla^2 w_z \quad ; \quad \therefore \underline{\frac{Dw_z}{Dt} = n\nabla^2 w_z}.$

If viscous effects are negligible, then $\frac{Dw_z}{Dt} = 0.$

Thus, for a planer flow, $\underline{w_z = \text{const}}$ if viscous effects are negligible.

8.71 a) $\vec{\nabla} \times \vec{V} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\hat{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\hat{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{k} = 0. \quad \therefore$ irrotational

$\frac{\partial f}{\partial x} = 10x. \quad \therefore f = 5x^2 + f(y)$

$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial y} = 20y. \quad \therefore f = 10y^2 + C. \quad$ Let $C = 0. \quad \therefore \underline{f = 5x^2 + 10y^2}$

b) $\vec{\nabla} \times \vec{V} = 0\hat{i} + 0\hat{j} + (8-8)\hat{k} = 0. \qquad \therefore$ irrotational

$\frac{\partial f}{\partial x} = 8y. \quad \therefore f = 8xy + f(y,z). \quad \frac{\partial f}{\partial y} = 8x + \frac{\partial f}{\partial y} = 8x. \quad \therefore \frac{\partial f}{\partial y} = 0$ and $f = f(z).$

$\frac{\partial f}{\partial z} = \frac{df}{dz} = -6z. \quad \therefore f = -3z^2 + C. \quad$ Let $C = 0.$

$\therefore \underline{f = 8xy - 3z^2}$

c) $\vec{\nabla}\times\vec{V} = 0\hat{i} + 0\hat{j} + \left( \frac{-y\frac{1}{2}(x^2+y^2)^{-1/2}2x}{x^2+y^2} - \frac{-x\frac{1}{2}(x^2+y^2)^{-1/2}2y}{x^2+y^2} \right)\hat{k} = 0.\quad \therefore \underline{\text{irrotational}}$

$\frac{\partial f}{\partial x} = \frac{x}{\sqrt{x^2+y^2}}.\qquad \therefore f = \sqrt{x^2+y^2} + f(y)$

$\frac{\partial f}{\partial y} = \frac{1}{2}(x^2+y^2)^{-1/2}2y + \frac{\partial f}{\partial y} = \frac{y}{\sqrt{x^2+y^2}}.\qquad \therefore \frac{\partial f}{\partial y} = 0.\quad \therefore f = C.\quad \text{Let } C = 0.$

$\therefore \underline{f = \sqrt{x^2+y^2}}$

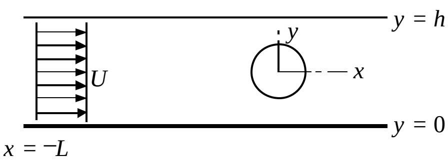
d) $\vec{\nabla}\times\vec{V} = 0\hat{i} + 0\hat{j} + \left[ \frac{-y(2x)}{(x^2+y^2)^2} - \frac{-x(2y)}{(x^2+y^2)^2} \right]\hat{k} = 0.\quad \therefore \underline{\text{irrotational}}$

$\frac{\partial f}{\partial x} = \frac{x}{x^2+y^2}.\qquad \therefore f = \frac{1}{2}\ell n(x^2+y^2) + f(y)$

$\frac{\partial f}{\partial y} = \frac{y}{x^2+y^2} = \frac{1}{2}\frac{2y}{x^2+y^2} + \frac{\partial f}{\partial y}.\qquad \therefore \frac{\partial f}{\partial y} = 0.\quad \therefore f = C.\quad \text{Let } C = 0.$

$\therefore \underline{f = \ell n\sqrt{x^2+y^2}}$

8.72 $\frac{\partial^2 y}{\partial x^2} + \frac{\partial^2 y}{\partial y^2} = 0.$ This requires two conditions on $x$ and two on $y$.




At $x = -L,\ u = U.\qquad \therefore \frac{\partial y}{\partial y} = U.$

At $x = L,\ u = U.\qquad \therefore \frac{\partial y}{\partial y} = U.$

At $y = -h,\ y = 0.$

At $y = h,\ y = U\times \text{h}.$ (See Example 8.9).

The boundary conditions are stated as:

$\underline{\frac{\partial y}{\partial y}(-L,y) = U,\ \frac{\partial y}{\partial y}(L,y) = U,\ y(x,-h) = 0,\ y(x,h) = 2Uh.}$

8.73 $u = \frac{\partial y}{\partial y} = 100.\quad \therefore y = 100y + f(x).\quad v = -\frac{\partial y}{\partial x} = -\frac{df}{dx} = 50.\quad \therefore f = -50x + C.$

$\therefore \underline{y(x,y) = 100y - 50x}.$ (We usually let $C = 0$.)

$u = \frac{\partial f}{\partial x} = 100.\quad \therefore f = 100x + f(y).\quad v = \frac{\partial f}{\partial y} = \frac{df}{dy} = 50.\quad \therefore f = 50y + C.$

$\therefore \underline{f(x,y) = 100x + 50y}.$

8.74 a) $\psi = 40\theta$.

b) $\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial \psi}{\partial \theta}\right) + \frac{1}{r}\left(-\frac{\partial^2 \psi}{\partial \theta \partial r}\right) = \frac{1}{r}\frac{\partial}{\partial r}(40) + \frac{1}{r}\left(-\frac{\partial}{\partial \theta}(0)\right) = 0.$

$\therefore$ It is incompressible since the above continuity equation is satisfied.
Note: The continuity equation is found in Table 5.1.

c) $\frac{\partial \phi}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{40}{r}. \quad \therefore \phi = 40\ell n r + f(\theta)$

$\frac{\partial \phi}{\partial \theta} = \frac{\partial f}{\partial \theta} = -r\frac{\partial \psi}{\partial r} = 0. \quad \therefore f = C.$ Let $C = 0$.

$\therefore \underline{\phi = 40\ell n\, r}$

d) $v_r = \frac{40}{r}, \quad v_\theta = 0. \quad a_r = v_r\frac{\partial v_r}{\partial r} = \frac{40}{r}\left(-\frac{40}{r^2}\right) = -10.$

$\therefore \underline{r = 5.43 \text{ m}}$

8.75 $u = \frac{\partial \psi}{\partial y} = 20\frac{2y}{x^2 + y^2} = \frac{\partial \phi}{\partial x}. \quad \therefore \phi = -40\tan^{-1}\frac{y}{x} + f(y).$

$v = \frac{\partial \phi}{\partial y} = -\frac{40\ /\ x}{1 + y^2\ /\ x^2} + \frac{\partial f}{\partial y} = -\frac{40x}{x^2 + y^2} + \frac{\partial f}{\partial y} = -20\frac{2x}{x^2 + y^2}. \quad \therefore f = C.$ Let $C = 0$.

$\underline{\phi = -40\tan^{-1}\frac{y}{x}.}$

8.76 a) $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0. \qquad \frac{\partial \psi}{\partial x} = 10y(x^2 + y^2)^{-2}(2x).$

$\frac{\partial^2 \psi}{\partial x^2} = 20y(x^2 - y^2)^{-2} - 80x^2y(x^2 + y^2)^{-3}$

$\frac{\partial \psi}{\partial y} = 10 - 10(x^2 + y^2)^{-1} + 10y(x^2 + y^2)^{-2}(2y).$

$\frac{\partial^2 \psi}{\partial y^2} = 20y(x^2 + y^2)^{-2} + 40y(x^2 + y^2)^{-2} - 80y^3(x^2 + y^2)^{-3}.$

$\therefore \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \frac{20y}{(x^2 + y^2)^2} - \frac{80x^2y}{(x^2 + y^2)^3} + \frac{60y}{(x^2 + y^2)^2} - \frac{80y^3}{(x^2 + y^2)^3}$

$= \frac{80y(x^2 + y^2)}{(x^2 + y^2)^3} - \frac{80x^2y}{(x^2 + y^2)^3} - \frac{80y^3}{(x^2 + y^2)^3} = \frac{80x^2y + 80y^3 - 80x^2y - 80y^3}{(x^2 + y^2)^3} = 0.$

b) In polar coord: $\psi(r, \theta) = 10r\sin\theta - \frac{10r\sin\theta}{r^2} = 10r\sin\theta - \frac{10}{r}\sin\theta.$

$\frac{1}{r}\frac{\partial \psi}{\partial \theta} = \left(10 - \frac{10}{r^2}\right)\cos\theta = \frac{\partial \phi}{\partial r}. \quad \therefore \phi = \left(10 + \frac{10}{r}\right)\cos\theta + f(\theta).$

$$\frac{1}{r}\frac{\partial\phi}{\partial\theta}=\frac{1}{r}\frac{df}{d\theta}-\left(10+\frac{10}{r^2}\right)\sin\theta=-\frac{\partial\psi}{\partial r}=-10\sin\theta-\frac{10}{r^2}\sin\theta.\ \ \frac{df}{d\theta}=0.\ f=\not{C}.$$

$$\therefore\ \phi=10\left(r+\frac{1}{r}\right)\cos\theta\quad\text{or}\quad \underline{\phi(x,y)=10x+\frac{10x}{x^2+y^2}},$$

where we let $r\cos\theta=x$ and $r^2=x^2+y^2$.

c) Along the $x$-axis, $v=-\dfrac{\partial\psi}{\partial x}=0$ where we let $y$ = 0 in part (a) and

$$u=\frac{\partial\psi}{\partial y}=10-\frac{10}{x^2+y^2}+\frac{20y^2}{(x^2+y^2)^2}=10-\frac{10}{x^2}\ \text{ with } y=0.$$

Euler's Eq: $\rho u\dfrac{\partial u}{\partial x}=-\dfrac{\partial p}{\partial x}.\quad \therefore\ \rho\left(10-\dfrac{10}{x^2}\right)\left(\dfrac{20}{x^3}\right)=-\dfrac{\partial p}{\partial x}.$

$$\therefore\ p=\int\rho\left(\frac{200}{x^5}-\frac{200}{x^3}\right)dx=\rho\left[-\frac{50}{x^4}+\frac{100}{x^2}\right]+C.\quad C=50\ 000.$$

$$\underline{=1000\left[\frac{100}{x^2}-\frac{50}{x^4}\right]+50\ 000\ \text{Pa.}}\quad \text{(Could have used Bernoulli!)}$$

d) Let $u=0$: $\quad 0=10-\dfrac{10}{x^2}.\quad \therefore\ x=\pm 1.\quad \therefore$ Stag pts: (1, 0), (–1, 0)

8.77 a) $$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=\frac{\partial}{\partial x}\left[10+\frac{10x}{x^2+y^2}\right]+\frac{\partial}{\partial y}\left[\frac{10y}{x^2+y^2}\right]=\frac{(x^2+y^2)10-10x(2x)}{(x^2+y^2)^2}$$

$$+\frac{(x^2+y^2)10-10y(2y)}{(x^2+y^2)^2}=\frac{10x^2+10y^2-20x^2+10x^2+10y^2-20y^2}{(x^2+y^2)^2}=0.$$

b) Polar coord: $\phi=10r\cos\theta+5\ell n\, r^2.$ (See Eq. 8.5.14.)

$$\frac{\partial\phi}{\partial r}=10\cos\theta+\frac{10r}{r^2}=\frac{1}{r}\frac{\partial\psi}{\partial\theta}.\quad \therefore\ \psi=10r\sin\theta+10\theta+f(r)$$

$$\frac{1}{r}\frac{\partial\phi}{\partial\theta}=-10\sin\theta=-\frac{\partial\psi}{\partial r}=-10\sin\theta-\frac{df}{dr}.\quad \therefore\ f=\not{C}.\quad \therefore\ \psi=10r\sin\theta+10\theta.$$

$$\therefore\ \underline{\psi(x,y)=10y+10\tan^{-1}\frac{y}{x}}.$$

c) $v=\dfrac{\partial\phi}{\partial y}=\dfrac{10y}{x^2+y^2}.\quad$ Along $x$-axis ($y$ = 0) $v$ = 0.

$$u=\frac{\partial\phi}{\partial x}=10+\frac{10x}{x^2+y^2}.\quad \text{Along } x\text{-axis } u=10+\frac{10}{x}.$$

Bernoulli: $\dfrac{V^2}{2}+\dfrac{p}{\rho}+\not{gz}=\dfrac{V_\infty^2}{2}+\dfrac{p_\infty}{\rho}+\not{gz_\infty}\quad$ (assume $z=z_\infty$)

$$\frac{(10+10/x)^2}{2}+\frac{p}{\rho}=\frac{10^2}{2}+\frac{100\ 000}{\rho}.\quad \therefore\ \underline{p=100-50\left(\frac{2}{x}+\frac{1}{x^2}\right)\text{ kPa.}}$$

d) $u = 0$: $\quad 0 = 10 + \dfrac{10}{x}. \quad \therefore x = -1. \quad \therefore$ Stag pt: $(-1,\ 0)$

e) $a_y = \cancel{v}\dfrac{\partial v}{\partial y} + u\cancel{\dfrac{\partial v}{\partial x}} = 0$ on $x-$axis. $\quad a_x = u\dfrac{\partial u}{\partial x} + \cancel{v}\dfrac{\partial u}{\partial y} = \left(10 + \dfrac{10}{x}\right)\left(-\dfrac{10}{x^2}\right).$

$\therefore a_x(-2, 0) = (10-5)\left(-\dfrac{10}{4}\right) = \underline{-12.5\ \text{m/s}^2}.$

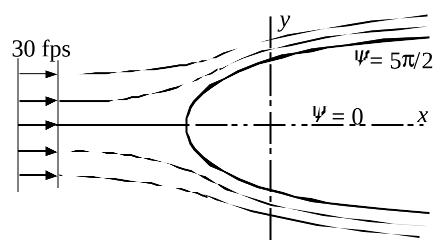
8.78 $\quad u(x,y) = y - 5y^2 = \dfrac{\partial \psi}{\partial y}. \quad \therefore \psi = \dfrac{y^2}{2} - \dfrac{5y^3}{3} + \cancel{C}. \quad \therefore \underline{\psi = \dfrac{1}{6}(3y^2 - 10y^3)}.$

$q = \int_0^{.2} u\,dy = \int_0^{.2}(y - 5y^2)dy = \dfrac{0.2^2}{2} - 5\dfrac{0.2^3}{3} = \underline{6.667\times10^{-3}\ \text{m}^2/\text{s}}.$

$\psi_2 - \psi_1 = \dfrac{1}{6}(3\times0.2^2 - 10\times0.2^3) - 0 = \underline{6.667\times10^{-3}\ \text{m}^2/\text{s}}.$

$\omega = -\dfrac{\partial u}{\partial y} = -1 + 10y \neq 0. \qquad \therefore \phi$ doesn't exist.

8.79 $\quad \psi = 30y + \dfrac{5\pi}{2\pi}\theta = 30r\sin\theta + \dfrac{5}{2}\theta.$




a) $v_r = \dfrac{1}{r}\dfrac{\partial \psi}{\partial \theta} = 30\cos\theta + \dfrac{5}{2r} = 0.$

At $\theta = \pi$, $\quad \dfrac{5}{2r_s} = 30. \quad \therefore r_s = 0.0833'.$

Stag. pt: $\quad \underline{(-1'', 0)}.$

b) At $\theta = \pi$, $r = .0833$, $\quad \psi_s = \dfrac{5\pi}{2} = 30r\sin\dfrac{\pi}{2} + \dfrac{5}{2}\dfrac{\pi}{2}.$

$\therefore r = y_{\text{inter}} = \underline{.0119\ \text{ft}}.$

c) $q = U \times H = \Delta\psi. \quad \therefore 30H = \dfrac{5\pi}{2}. \quad \therefore H = \dfrac{5\pi}{60}.$ Thickness $= 2H = \dfrac{5\pi}{30}$ ft or $\underline{1.257''}$.

d) $v_r(1,\pi) = 30\cos\pi + \dfrac{5}{2} = -30 + 2.5 = -27.5. \quad \therefore \underline{u = 27.5\ \text{fps}}.$

8.80 $\quad \phi = \dfrac{\pi}{2\pi}\ell n\left[(x+1)^2 + y\right]^{1/2} - \dfrac{\pi}{2\pi}\ell n\left[(x-1)^2 + y^2\right]^{1/2} + 10x = \dfrac{1}{4}\ell n\left[(x+1)^2 + y^2\right]$

$$-\dfrac{1}{4}\ell n\left[(x-1)^2 + y^2\right] + 10x.$$

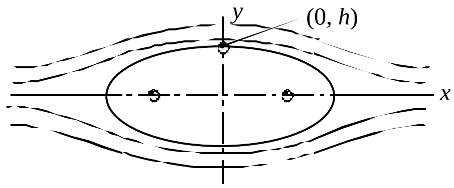
$$u = \left.\dfrac{\partial \phi}{\partial x}\right|_{y=0} = \dfrac{\frac{1}{4}[2(x+1)]}{(x+1)^2} - \dfrac{\frac{1}{4}[2(x-1)]}{(x-1)^2} + 10 = \dfrac{1}{2(x+1)} - \dfrac{1}{2(x-1)} + 10. \quad v = 0 \text{ if } y = 0.$$

At the stagnation point, $u = 0$. $\therefore \dfrac{1}{2(x+1)} - \dfrac{1}{2(x-1)} + 10 = 0$. $\therefore \dfrac{2}{x^2 - 1} = 20$.

$\therefore x^2 = 1.1$. $\therefore x = \pm 1.049$ m. $\therefore$ oval length $= 2 \times 1.049 = \underline{2.098 \text{ m}}$.

All the flow from the source goes to the sink, i.e., $p$ m$^2$ / s, or $\dfrac{p}{2}$ m$^2$ / s for $y > 0$.

$$u(y) = \left.\frac{¶f}{¶x}\right|_{x=0} = \frac{\frac{1}{4}(2)}{1+y^2} - \frac{\frac{1}{4}(-2)}{1+y^2} + 10 = \frac{1}{1+y^2} + 10.$$




$$q = \int_0^h \left( \frac{1}{1+y^2} + 10 \right) dy = \frac{p}{2}. \quad \therefore \tan^{-1} h + 10h = \frac{p}{2}.$$

$h = 0.143$ m so that thickness $= 2h = \underline{0.286 \text{ m.}}$

The minimum pressure occurs on the oval surface at $(0,h)$.

There $u = \dfrac{1}{1+.143^2} + 10 = 10.98$ m / s.

Bernoulli: $\dfrac{V^2}{2} + \dfrac{p}{r} = \dfrac{V_\infty^2}{2} + \dfrac{p_\infty}{r}$. $\dfrac{10.98^2}{2} + \dfrac{p}{1000} = \dfrac{10^2}{2} + \dfrac{10\,000}{1000}$.

$$\therefore \underline{p_{\min} = -280 \text{ Pa}}.$$

8.81 $$f = \frac{2p}{2p} \ell n\left[(x+1)^2 + y^2\right]^{1/2} + \frac{-2p}{2p} \ell n\left[(x-1)^2 + y^2\right]^{1/2} + 2x = \frac{1}{2} \ell n\left[(x+1)^2 + y^2\right]$$
$$- \frac{1}{2} \ell n\left[(x-1)^2 + y^2\right] + 2x.$$

$$u = \frac{¶f}{¶x} = \frac{\frac{1}{2}2(x+1)}{(x+1)^2 + y^2} - \frac{\frac{1}{2}2(x-1)}{(x-1)^2 + y^3} + 2. \quad v = \frac{y}{(x-1)^2 + y^2} - \frac{y}{(x-1)^2 + y^2}$$

Along the $x$-axis ($y = 0$), $v = 0$ and $u = \dfrac{1}{x+1} - \dfrac{1}{x-1} + 2$.

Set $u = 0$: $\dfrac{1}{x-1} - \dfrac{1}{x+1} = 2$, or $x^2 = 2$. $\therefore x = \pm\sqrt{2}$.

Stag. pts.: $\underline{(\sqrt{2},0),\ (-\sqrt{2},0)}$.

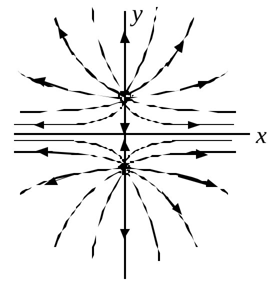
$u(4,0) = \dfrac{1}{-4+1} - \dfrac{1}{-4-1} + 2 = \underline{1.867 \text{ m / s}}$. $v(-4,0) = \underline{0}$.

$u(0,4) = \dfrac{1}{1+4^2} - \dfrac{-1}{1+4^2} + 2 = \underline{2.118 \text{ m / s}}$. $v(0,4) = \dfrac{4}{1+4^2} - \dfrac{4}{1+4^2} = \underline{0}$.

8.82 $$f = \frac{2p}{2p} \ell n[x^2 + (y-1)^2]^{1/2} + \frac{2p}{2p} \ell n[x^2 + (y+1)^2]^{1/2}$$
$$= \frac{1}{2} \ell n[x^2 + (y-1)^2] + \frac{1}{2} \ell n[x^2 + (y+1)^2].$$

$$u = \frac{\partial \phi}{\partial x} = \frac{x}{x^2 + (y-1)^2} + \frac{x}{x^2 + (y+1)^2}.$$

$$v = \frac{\partial \phi}{\partial y} = \frac{y-1}{x^2 + (y-1)^2} + \frac{y+1}{x^2 + (y-1)^2}.$$




At (0, 0) $u = 0$ and $v = 0$. At (1, 1)

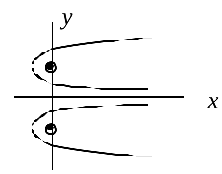
$$v = 0 + \frac{2}{2^2 + 1^2} = \frac{2}{5} = 0.4 \text{ m/s.} \quad u = \frac{1}{1^2} + \frac{1}{2^2 + 1^2} = 1.2 \text{ m/s.}$$

$$\therefore \underline{\vec{V} = 1.2\hat{i} + 0.4\hat{j} \text{ m/s}}.$$

8.83 $$\phi = \frac{2\pi}{2\pi}\ell n[(y-1)^2 + x^2]^{1/2} + \frac{2\pi}{2\pi}\ell n[(y+1)^2 + x^2]^{1/2} + U_\infty x.$$

$$= \frac{1}{2}\ell n[(y-1)^2 + x^2] + \frac{1}{2}\ell n[(y+1)^2 + x^2] + U_\infty x.$$

a) Stag. pts. May occur on $x$-axis, $y = 0$.




$$u = \left.\frac{\partial \phi}{\partial x}\right|_{y=0} = \frac{x}{1+x^2} + \frac{x}{1+x^2} + 10.$$

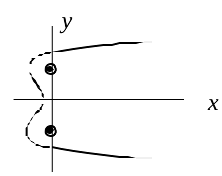
$\therefore x^2 + 0.2x + 1 = 0.$ $\therefore$ no stagnation points exist on the $x$-axis.

(They do exist away from the $x$-axis.)

Along the $y$-axis: $u(y) = 10.$ $q = \int_0^h u\,dy = \frac{1}{2}(2\pi) = \pi$ m$^2$/s.

$$\therefore \pi = \int_0^h 10\,dy = 10\,h. \qquad \therefore h = \underline{0.314 \text{ m}}.$$

b) $u = \frac{2x}{1+x^2} + 1.$ $\therefore x^2 + 2x + 1 = 0.$ $\therefore x = -1$ m.




Stag. pt.: <u>(–1, 0)</u>

Along the $y$-axis: $u = 1.0.$ $\therefore \pi = 1 \times h.$ $\therefore \underline{h = 3.14 \text{ m}}.$

c) $u = \frac{2x}{1+x^2} + 0.2.$ $\therefore x^2 + 10x + 1 = 0.$ $\therefore x = -9.90,\ -0.10$ m.

Stag. pts.: <u>(–9.9, 0)</u> , <u>(–0.1, 0)</u>.

Along the $y$-axis: $u = 0.2.$ $\therefore \pi = 0.2\,h.$ $\therefore \underline{h = 15.71 \text{ m}}.$

8.84 $$\phi = \frac{60}{r}\cos\theta + 8r\cos\theta.$$

a) $$v_r = \frac{\partial \phi}{\partial r} = -\frac{60}{r^2}\cos\theta + 8\cos\theta = \left(8 - \frac{60}{r^2}\right)\cos\theta.$$

At the cylinder surface $v_r = 0$ for all $\theta$. Hence,

$$\frac{60}{r_c^2} = 8. \qquad \therefore\ \underline{r_c = 2.739\ \text{m}}$$

b) Bernoulli: $\Delta p = \rho \frac{U_\infty^2}{2} = 1000 \frac{8^2}{2} = 32\ 000\ \text{Pa}$ or $\underline{32\ \text{kPa}}$

c) $v_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\frac{60}{r^2}\sin\theta - 8\sin\theta.$

At $r = r_c$, $v_\theta = -8\sin\theta - 8\sin\theta = \underline{-16\sin\theta}$

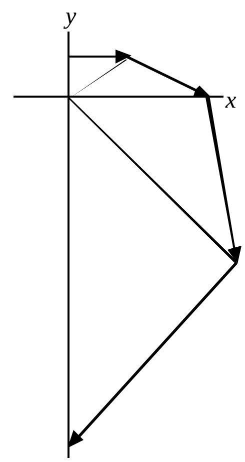
d) $\Delta p = \rho \frac{v_{90^\circ}^2}{2} = 1000 \frac{16^2}{2} = 128\ 000\ \text{Pa}$ or $\underline{128\ \text{kPa}}$

8.85 $\psi = \frac{4\pi}{2\pi}\theta + \frac{20\pi}{2\pi}\ell n\, r = 2\theta + 10\ell n\, r$

At $(x,y) = (0,1)$, $(r,\theta) = (1, \pi/2)$.

$$v_r(1,\pi/2) = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{1}{1}(2) = 2.$$

$$v_\theta(1,\pi/2) = -\frac{\partial \psi}{\partial r} = -\frac{10}{1} = -10.$$




$v_r(1.7,\pi/4) = \frac{2}{1.7} = 1.18$, $\quad v_\theta(1.7,\pi/4) = \frac{-10}{1.7} = -5.88$

$v_r(3.2,0) = \frac{2}{3.2} = 0.625$, $\quad v_\theta(3.2,0) = \frac{-10}{3.2} = 3.125$

$v_r(6,-\pi/4) = \frac{2}{6} = 0.333$, $\quad v_\theta(6,-\pi/4) = \frac{-10}{6} = -1.67$, etc.

Note: We scaled the radius at each 45° increment to find $r$.

b) $v_r = \frac{2}{r}$ and $v_\theta = -\frac{10}{r}$. From Table 5.1 (use the l.h.s. of momentum)

$$a_r = \frac{Dv_r}{Dt} + \frac{v_\theta^2}{r} = v_r\frac{\partial v_r}{\partial r} - \frac{v_\theta^2}{r} = \frac{2}{r}\left(-\frac{2}{r^2}\right) - \frac{100}{r^3} = -\frac{104}{1}$$

$$= -104\ \text{m/s}^2$$

$$a_\theta = \frac{Dv_\theta}{Dt} + \frac{v_r v_\theta}{r} = v_r\frac{\partial v_\theta}{\partial r} + \frac{v_r v_\theta}{r} = \frac{2}{r}\left(\frac{10}{r^2}\right) + \frac{2(-10)}{r^3} = 0.$$

$$\therefore \vec{a}(0,1) = \underline{(-104, 0)\ \text{m/s}^2}$$

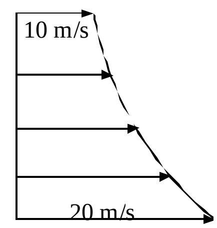
c) $v_r(14.14, p\ /\ 4) = 2\ /\ 14.14 = 0.1414, \quad v_q(14.14, p\ /\ 4) = -\frac{10}{14.14} = -0.707$

$v_r(0.1, p\ /\ 2) = 2\ /\ 0.1 = 20, \quad v_q(0.1, p\ /\ 2) = -10\ /\ 0.1 = -100.$

Bernoulli: $\frac{20\ 000}{1.2} + \frac{0.1414^2 + 0.707^2}{2} = \frac{p}{1.2} + \frac{20^2 + 100^2}{2}. \quad \therefore \underline{p = 13\ 760\ \text{Pa}}$

We used $r_{\text{air}} = 1.2\ \text{kg}\ /\ \text{m}^3$ at standard conditions.

8.86 Along the $y$-axis $v_r = 0$ and $v_q = -10 - \frac{40}{r^2}$.

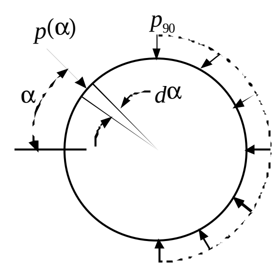



We have set $q = \frac{p}{2}$ in Eq. 8.5.27. $r_c = \sqrt{\frac{40}{10}} = 2.$

b) $v_r = 10\cos q - \frac{40}{r^2}\cos q. \quad (-4,3) \Rightarrow (5, 126.9^\circ).$

$v_q = -10\sin q - \frac{40}{r^2}\sin q. \quad \therefore \underline{v_r = -6.96\ \text{m}\ /\ \text{s}}, \ \underline{v_q = -9.28\ \text{m}\ /\ \text{s}}.$

c) Use Eq. 8.5.28: $p = p_0 - 2rU_\infty^2 \sin^2 a$



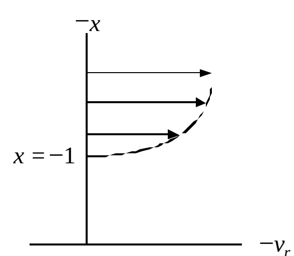

$$\text{Drag} = \int_{-p/2}^{p/2} p\cos a\ r_c daL - p_{90} \times 2r_cL. \quad p_{90} = p_0 - 2rU_\infty^2.$$

$$= 2\int_0^{p/2} (p_0 - 2rU_\infty^2 \sin^2 a)\cos a\ r_cLda - p_{90} \times 2r_cL$$

$$= 2r_cL\left[p_0 \sin a - 2rU_\infty^2 \frac{\sin^3 a}{3}\right]_0^{p/2} - \left[p_0 - 2rU_\infty^2\right]2r_cL = \frac{8}{3}r_cLrU_\infty^2.$$

$$C_D = \frac{\text{Drag}}{\frac{1}{2}rU_\infty^2 A} = \frac{(8\ /\ 3)r_cLrU_\infty^2}{\frac{1}{2}rU_\infty^2 2r_cL} = \frac{8}{3} = \underline{2.667}.$$

8.87 $v_r = U_\infty \cos q - \frac{m}{r^2}\cos q$. Let $U_\infty = 4,\ m = r_c^2 U_\infty = 1^2 \times 4 = 4.$




For $q = p,\ v_r = -4 + \frac{4}{r^2}.$

b) $v_q = -\frac{¶y}{¶r} = -U_\infty \sin q - \frac{m\sin q}{r_c^2} = \left(-4 - \frac{4}{1^2}\right)\sin q = \underline{-8\sin q}.$

c) $p_c = p_\infty + r\frac{V_\infty^2}{2} - r\frac{v_q^2}{2} = 50\ 000 + 1000 \times \frac{4^2}{2} - 1000\frac{8^2 \sin^2 q}{2}.$

$$\therefore p_c = \underline{58 - 32\sin^2 q\ \ \text{kPa.}}$$

d) Drag $= 2\int_0^{\pi/2}(58 - 32\sin^2\alpha)\cos\alpha \times 1 \times 1 d\alpha - 26 \times 2 \times 1$

$$= 2\left[58 - 32\left(\frac{1}{3}\right)\right] - 52 = \underline{42.7 \text{ kN}}.$$ (See the figure in Problem 8.86c.)

8.88 On the cylinder $v_\theta = -2U_\infty \sin\theta - \dfrac{\Gamma}{2\pi r_c} = -60\sin\theta - \dfrac{1000}{2\pi \times 3.651}$, where we have

Used $r_c = \sqrt{\dfrac{m}{U_\infty}} = \sqrt{\dfrac{400}{30}} = 3.651$ ft.

If

$$u(x,y) = -.0318\left[\frac{x-6}{(x-6)^2+(y-2)^2} + \frac{x-6}{(x-6)^2+(y+2)^2} + \frac{x+6}{(x+6)^2+(y-2)^2} + \frac{x+6}{(x+6)^2+(y+2)^2}\right]$$

$\therefore \theta = 227^\circ, 313^\circ.$

Stag. pts.: $\underline{(3.651 \text{ ft}, 227^\circ)}$ , $\underline{(3.651 \text{ ft}, 313^\circ)}$.

Max. pressure occurs on the cylinder at a stagnation pt.:

$$\therefore p_{\max} = \frac{\rho}{2}\left[U_\infty^2 - v_o^2\right] = \frac{0.0024}{2}\left[30^2 - 0^2\right] = \underline{1.08 \text{ psf}}.$$

Min. pressure occurs at the top of the cylinder where $\theta = 90^\circ$ and the velocity is:

$$v_{90} = -2U_\infty \sin\theta - \frac{\Gamma}{2\pi r_o} = -2 \times 30 - \frac{1000}{2\pi \times 3.651} = 104 \text{ fps}$$

$$\therefore p_{\min} = \frac{\rho}{2}\left[U_\infty^2 - v_o^2\right] = \frac{0.0024}{2}\left[30^2 - 104^2\right] = \underline{-11.8 \text{ psf}}.$$

8.89 $v_\theta = -2 \times 20 \sin\theta - \dfrac{\Gamma}{2\pi \times .4}$. For one stag. pt.: $v_\theta = 0$ at $\theta = 270^\circ$:

$$0 = -2 \times 20 \sin 270^\circ - \frac{\Gamma}{2\pi \times .4}. \quad \therefore \Gamma = 2 \times 20 \times 2\pi \times .4 = 100.5 \text{ m}^2/\text{s}.$$

$$\Gamma = 2\pi r_c^2 \omega. \quad \therefore \omega = \frac{\Gamma}{2\pi r_c^2} = \frac{100.5}{2\pi \times .4^2} = \underline{100 \text{ rad/s}}.$$ (See Example 8.12.)

Min. pressure occurs where $|v_\theta|$ is max, i.e., $\theta = \pi/2$. There

$$v_\theta = -2 \times 20 \times 1 - \frac{100.5}{2\pi \times .4} = 80 \text{ m/s}.$$

$$\therefore p_{\min} = p_\infty + \frac{V_\infty^2}{2}\rho - \frac{v_\theta^2}{2}\rho = 0 + \frac{20^2}{2} \times 1.22 - \frac{80^2}{2} \times 1.22 = \underline{-3660 \text{ Pa}}.$$

8.90 $\Gamma = 2pr_c^2 w = 2p \times .6^2 \times 120 \times 2p\ /60 = 28.42\ \text{m}^2\ /\text{s}.\ m = r_c^2 U = .6^2 \times 3 = 1.08\ \text{m}^3\ /\text{s}.$

$\therefore v_q = -2 \times 3 \sin q - \dfrac{28.42}{2p \times .6}.\quad \therefore \sin q = -1.256.$ Impossible. $\therefore$ Stag. pt. is off the cylinder at $q = 270^\circ$, but $r > r_c$. From Eq. 8.5.29,

$$v_q = -\frac{¶y}{¶r} = -U_\infty \sin q - \frac{m}{r^2}\sin q - \frac{\Gamma}{2pr} = -3(-1) - \frac{1.08}{r^2}(-1) - \frac{28.42}{2pr} = 0.$$

$$\therefore\ 3 + \frac{1.08}{r^2} = \frac{4.523}{r}.\quad \therefore\ r^2 - 1.508r + 0.36 = 0.\quad \therefore\ r = 1.21\ \text{m}.$$

Stag. pt.: <u>(1.21, 270°)</u>. $\quad (v_q)_{90^\circ} = -2 \times 3 - \dfrac{28.42}{2p \times .6} = -13.54\ \text{m/s}.$

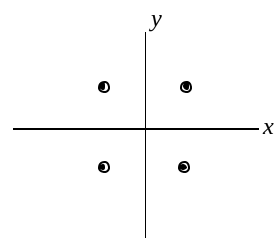
Min. pressure occurs at $q = 90^\circ$, at $r = r_c$: $p_{\min} = \left(\dfrac{3^2}{2} - \dfrac{13.54^2}{2}\right)1.22 = \underline{-106\ \text{Pa}}.$

Max. pressure occurs at $q = 270^\circ$, at $r = r_c$: $p_{\max} = \left(\dfrac{3^2}{2} - \dfrac{1.54^2}{2}\right)1.22 = \underline{-4.04\ \text{Pa}}.$

8.91 At 15,000 ft, $r = .0015\ \text{slug}\ /\text{ft}^3$.

Lift = $rU_\infty \Gamma L = .0015 \times 350 \times 15{,}000 \times 60 = \underline{472{,}000\ \text{lb}}.$

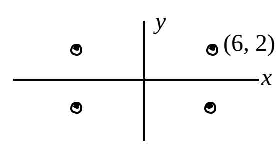
8.92 Place four sources as shown. Then, with $q = 2p$ for each:




$$u(x,y) = \frac{x-2}{(x-2)^2 + (y-2)^2} + \frac{x+2}{(x+2)^2 + (y-2)^2} + \frac{x-2}{(x-2)^2 + (y+2)^2} + \frac{x+2}{(x+2)^2 + (y+2)^2}$$

$$v(x,y) = \frac{y-2}{(x-2)^2 + (y-2)^2} + \frac{y+2}{(x-2)^2 + (y+2)^2} + \frac{y-2}{(x+2)^2 + (y-2)^2} + \frac{y+2}{(x+2)^2 + (y+2)^2}$$

8.93 Place four sources with $q = 0.2\ \text{m}^2\ /\text{s}$, as shown.




$$u(x,y) = -.0318\left[\frac{x-6}{(x-6)^2 + (y-2)^2} + \frac{x-6}{(x-6)^2 + (y+2)^2} + \frac{x+6}{(x+6)^2 + (y-2)^2} + \frac{x+6}{(x+6)^2 + (y+2)^2}\right]$$

$$v(x,y) = -.0318\left[\frac{y-2}{(x-6)^2 + (y-2)^2} + \frac{y-2}{(x+6)^2 + (y-2)^2} + \frac{y+2}{(x-6)^2 + (y+2)^2} + \frac{y+2}{(x+6)^2 + (y+2)^2}\right]$$

where $\dfrac{q}{2p} = \dfrac{-.2}{2p} = -.0318.$

At (4,3) $u(4,3)=-.0318\left[\frac{-2}{4+1}+\frac{-2}{4+25}+\frac{10}{100+1}+\frac{10}{100+25}\right]=\underline{0.00922\ \text{m / s}}.$

$$v(4,3)=-.0318\left[\frac{1}{4+1}+\frac{1}{100+1}+\frac{5}{4+25}+\frac{5}{100+25}\right]=\underline{-0.01343\ \text{m / s}}.$$

8.94 $\text{Re}_{\text{crit}}=\frac{U_\infty x_T}{\nu}.\quad \therefore x_T=6\times10^5\nu\ /\ 300=2000\nu.$

a) $\nu=1.56\times10^{-4}\ \text{ft}^2\ /\ \text{sec}.\quad \therefore x_T=2000\times1.56\times10^{-4}=0.312'$ or $\underline{3.74''}$

b) $\nu=\frac{\mu}{\rho}=2.1\times10^{-4}\ \text{ft}^2\ /\ \text{sec}.\quad \therefore x_T=2000\times2.1\times10^{-4}=0.42'$ or $\underline{5.04''}$

c) $\nu=3.47\times10^{-4}\ \text{ft}^2\ /\ \text{sec}.\quad \therefore x_T=2000\times3.47\times10^{-4}=0.694'$ or $\underline{8.33''}$

8.95 a) Use $\text{Re}_{\text{crit}}=3\times10^5=10x_T\ /\ 1.51\times10^{-5}.\quad \therefore x_T=\underline{0.453\ \text{m}}.$

b) Use $\text{Re}_{\text{crit}}=10^6=10x_T\ /\ 1.51\times10^{-5}.\quad \therefore x_T=\underline{1.51\ \text{m}}.$

c) Use $\text{Re}_{\text{crit}}=3\times10^5=10x_T\ /\ 1.51\times10^{-5}.\quad \therefore x_T=\underline{0.453\ \text{m}}.$

d) Use $\text{Re}_{\text{crit}}=3\times10^5=10x_T\ /\ 1.51\times10^{-5}.\quad \therefore x_T=\underline{0.453\ \text{m}}.$

e) $\text{Re}=6\times10^4=10x_{\text{growth}}/1.51\times10^{-5}.\quad \therefore x_{\text{growth}}=0.091\ \text{m}$ or $\underline{9.1\ \text{cm}}$.

Note: A rough plate, high free-stream disturbances, or a vibrated smooth plate all experience transition at the lower $\text{Re}_{\text{crit}}$.

8.96 a) Use $\text{Re}_{\text{crit}}=3\times10^5=10x_T\ /\ 10^{-6}.\quad \therefore x_T=0.03\ \text{m}$ or $\underline{3\ \text{cm}}$.

b) Use $\text{Re}_{\text{crit}}=10^6=10x_T\ /\ 10^{-6}.\quad \therefore x_T=0.1\ \text{m}$ or $\underline{10\ \text{cm}}$.

c) Use $\text{Re}_{\text{crit}}=3\times10^5=10x_T\ /\ 10^{-6}.\quad \therefore x_T=0.03\ \text{m}$ or $\underline{3\ \text{cm}}$.

d) Use $\text{Re}_{\text{crit}}=3\times10^5=10x_T\ /\ 10^{-6}.\quad \therefore x_T=0.03\ \text{m}$ or $\underline{3\ \text{cm}}$.

e) $p(x)=20\ 000-2\ \times1000\ \times10^2\sin^2(x/2)$

8.97 $\text{Re}_{\text{crit}}=6\times10^5=\frac{U_\infty\times2}{\nu}.$ For a wind tunnel: $6\times10^5=\frac{U_\infty\times2}{1.5\times10^{-5}}.$

$$\therefore \underline{U_\infty=4.5\ \text{m / s}}.$$

For a water channel: $6\times10^5=\frac{U_\infty\times2}{10^{-6}}.\quad \therefore \underline{U_\infty=0.3\ \text{m / s}}.$

8.98 The $x$-coordinate is measured along the cylinder surface as shown in Fig. 8.19. The pressure distribution (see solution 8.86) on the surface is $p=p_0-2\rho U_\infty^2\sin^2\alpha$ where $r_c\alpha=x$ ($\alpha$ is zero at the stagnation point). Then

$$p(x)=20\ 000-2\ \times1000\ \times10^2\sin^2(x/2)$$
$$=\underline{20-200\sin^2(x\ /\ 2)\ \text{kPa}}$$

The velocity *U(x)* at the edge of the b.l. is *U(x)* on the cylinder wall:

$$v_q(r=2) = -10\sin q - 10\sin q = -20\sin(p - a) = 20\sin a$$

$$\therefore U(x) = \underline{20\sin(x/2)}$$

8.99 $U(x) = v_q$ at $r_c = 1$. $v_q = 8\sin a$. $\therefore U(x) = \underline{8\sin x}$ since $x = a\, r_c$.

$$p(x) = 58 - 32\sin^2 a = \underline{58 - 32\sin^2 x \text{ kPa}}$$

8.100 The height $h$ above the plate is $h(x) = mx + .4$. $.1 = m \times 2 + .4$ $\therefore m = -.15$

$\therefore h(x) = 0.4 - .15x$. Continuity: $6 \times .4 = U(x)h$. $\therefore U(x) = \dfrac{2.4}{0.4 - .15x}$ or

$$\underline{U(x) = \frac{16}{2.67 - x}}.$$

Euler's Eqn: $ru\dfrac{¶u}{¶x} = -\dfrac{¶p}{¶x}$. $\therefore \dfrac{dp}{dx} = r\dfrac{16}{2.67 - x}\dfrac{16}{(2.67 - x)^2}$

$$= \underline{\frac{256}{(2.67 - x)^3}}.$$

8.101 a) $\dot{m}_{\text{top}} = \dot{m}_{\text{out}} - \dot{m}_{\text{in}} = \int_0^d r u dy + \dfrac{\partial}{\partial x}\int_0^d r u dy dx - \int_0^d r u dy = \dfrac{\partial}{\partial x}\int_0^d r u dy dx$

b) $\Sigma F_x = pd - t_0 dx + (p + \dfrac{dp}{2})dd - (p + dp)(d + dd)$

$$= -t_0 dx - d\, dp + \text{~~higher order terms~~}$$

$$m\dot{o}m_{\text{out}} - m\dot{o}m_{\text{in}} - m\dot{o}m_{\text{top}} = \int_0^d r u^2 dy + \frac{\partial}{\partial x}\int_0^d r u^2 dy dx - \int_0^d r u^2 dy - U(x)\left(\frac{\partial}{\partial x}\int_0^d r u dy dx\right)$$

$$= \frac{\partial}{\partial x}\int_0^d r u^2 dy dx - U(x)\left(\frac{\partial}{\partial x}\int_0^d r u dy dx\right)$$

$$= 4\left[-\frac{3 \times 4}{2 \times 4.65^2 \times 1.5 \times 10^{-5} \times 3}y + \frac{3}{2}\frac{4^2 y^3}{4.65^4 \times (1.5 \times 10^{-5} \times 3)^2}\right]\frac{dd}{dx}.$$

8.102 $t_0 = -d\dfrac{dp}{dx} + U(x)\dfrac{d}{dx}\int_0^d r u dy - \dfrac{d}{dx}\int_0^d r u^2 dy$

$$= -d\frac{dp}{dx} + \frac{d}{dx}\int_0^d r u U dy - \frac{dU}{dx}\left(\int_0^d r u dy\right) - \frac{d}{dx}\int_0^d r u^2 dy$$

where we have used $g\dfrac{df}{dx} = \dfrac{dfg}{dx} - f\dfrac{dg}{dx}$. $\left(\text{Here } U = g,\ f = \int_0^d r u dy.\right)$

$$\therefore t_0 = -d\frac{dp}{dx} + \frac{d}{dx}\int_0^d r u(U - u)dy - r\frac{dU}{dx}\int_0^d u dy. \qquad (r = \text{const.})$$

8.103 $\frac{dp}{dx} = -\frac{r}{2}\frac{d}{dx}U^2 = -rU\frac{dU}{dx} = -r\frac{dU}{dx}\left(\frac{1}{d}\int_0^d Udy\right)$ where $U = \frac{1}{d}\int_0^d Udy$.

$$\therefore t_0 = -d\left[-r\frac{dU}{dx}\left(\frac{1}{d}\int_0^d Udy\right)\right] + r\frac{d}{dx}(qU^2) - r\frac{dU}{dx}\int_0^d udy$$

$$= r\frac{d}{dx}(qU^2) + r\frac{dU}{dx}\int_0^d (U-u)dy = r\frac{d}{dx}(qU^2) + r\frac{dU}{dx}Ud_d.$$

8.104 If $dp/dx = 0$ then $\frac{dU}{dx} = 0$ and $t_0 = r\frac{d}{dx}\int_0^d u(U_\infty - u)dy$.

$$t_0 = r\frac{d}{dx}\int_0^d U_\infty^2 \sin\frac{py}{2d}\left(1 - \sin\frac{py}{2d}\right)dy = rU_\infty^2\frac{d}{dx}\left[-\frac{2d}{p}\cos\frac{py}{2d} - \frac{y}{2}\right]_0^d = rU_\infty^2\frac{d}{dx}\left(\frac{2d}{p} - \frac{d}{2}\right)$$

$$t_0 = m\left.\frac{\P u}{\P y}\right|_{y=0} = mU_\infty\frac{p}{2d}\cos 0.$$

$$\therefore mU_\infty\frac{p}{2d} = .137\, rU_\infty^2\frac{dd}{dx}. \quad \therefore ddd = 11.5\frac{n}{U_\infty}dx. \quad \therefore \underline{d = 4.79\sqrt{\frac{nx}{U_\infty}}}.$$

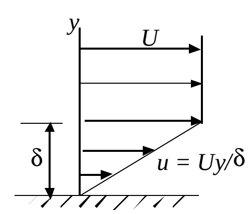
b) $t_0 = mU_\infty\frac{p}{2}\frac{1}{4.79}\sqrt{\frac{U_\infty}{nx}} = \underline{0.328mU_\infty\sqrt{\frac{U_\infty}{nx}}}$.

c) $\frac{\P u}{\P x} = U_\infty\frac{\P}{\P x}\sin\left(\frac{py}{2\times 4.79}\sqrt{\frac{U_\infty}{nx}}\right) = U_\infty\frac{\P}{\P x}\sin\left(\frac{a}{\sqrt{x}}\right) = U_\infty\left(-\frac{ax^{-3/2}}{2}\right)\cos\frac{a}{\sqrt{x}} = -\frac{\P v}{\P y}$.

$$\therefore v = \int_0^d U_\infty\frac{.164y}{x^{3/2}}\sqrt{\frac{U_\infty}{n}}\cos\left(.328y\sqrt{\frac{U_\infty}{nx}}\right)dy = \underline{\int_0^d .0316U_\infty\sqrt{\frac{U_\infty}{n}}\cos\left[\left(.189\sqrt{\frac{U_\infty}{n}}\right)y\right]dy}.$$

8.105 $u = U_\infty\frac{y}{d}$. $\quad t_0 = r\frac{d}{dx}\int_0^d U_\infty^2\frac{y}{d}\left(1 - \frac{y}{d}\right)dy$

$$= r\frac{d}{dx}U_\infty^2\left(\frac{d}{2} - \frac{d}{3}\right) = \frac{1}{6}rU_\infty^2\frac{dd}{dx}.$$




$$t_0 = m\frac{\P u}{\P y} = m\frac{U_\infty}{d}. \quad \therefore m\frac{U_\infty}{d} = \frac{1}{6}rU_\infty^2\frac{dd}{dx}. \quad \therefore ddd = 6\frac{n}{U_\infty}dx$$

$$\therefore d^2 = 12\frac{n}{U_\infty}x. \quad d(x) = 3.46\sqrt{\frac{nx}{U_\infty}}. \quad t_0 = 0.289mU_\infty\sqrt{\frac{U_\infty}{nx}}.$$

%error in $d(x) = \frac{5 - 3.46}{5}\times 100 = \underline{30.8\%\text{ low}}.$

$\%\text{error in } t_0(x) = \dfrac{.332-.289}{.332} \times 100 = \underline{13\% \text{ low}}.$

8.106 $t_0 = \dfrac{d}{dx}\left\{ \int_0^{d/6} r\,3U_\infty^2 \dfrac{y}{d}\left(1-3\dfrac{y}{d}\right)dy + \int_{d/6}^{d/2} rU_\infty^2\left(\dfrac{y}{d}+\dfrac{1}{3}\right)\left(1-\dfrac{y}{d}-\dfrac{1}{3}\right)dy \right.$

$$\left. +\int_{d/2}^{d} rU_\infty^2\left(\frac{y}{3d}+\frac{2}{3}\right)\left(1-\frac{y}{3d}-\frac{2}{3}\right)dy \right\}$$

$$= \frac{d}{dx} rU_\infty^2(0.1358d) = m\frac{3U_\infty}{d}. \qquad \therefore d\frac{dd}{dx} = 22.08m\ /\ rU_\infty.$$

Thus, $d(x) = 6.65\sqrt{vx/U_\infty},\ t_0(x) = 0.1358\,rU_\infty^2\left(\dfrac{6.65}{2}\sqrt{\dfrac{v}{U_\infty x}}\right) = \underline{0.451\,rU_\infty^2\,\text{Re}_x^{-1/2}}.$

$\%\text{error for } d = \dfrac{6.65-5}{5}\times 100 = \underline{33\%}.$ $\quad \%\text{error for } t_0 = \dfrac{0.451-0.332}{0.332}\times 100 = \underline{36\%}$

8.107 Continuity from entrance to $x$: $\quad U_0H = 2\int_0^d u(y)dy + U(x)(H-2d).$

Write $U(x)d = U(x)\int_0^d dy = \int_0^d U(x)dy.$ Then, continuity provides

$$U_0H = 2\int_0^d (u-U)dy + UH = UH - 2\int_0^d (U-u)dy$$

$$= UH - 2Ud_d. \qquad \therefore U(x) = \frac{U_0H}{H-2d_d}.$$

If we were to move the walls out a distance $d_d(x)$, then $U(x)$ would be constant since $\left[(H-2d_d)+2d_d\right]$ would be constant; then $U(x) = U_0$. For a square wind tunnel, displace one wall outward $4d_d$ for $dp\ /\ dx = 0$.

8.108 The given velocity profile is that used in Example 8.13. There we found $d = 5.48\sqrt{nx\ /\ U_\infty} = 5.48\sqrt{10^{-6}x\ /\ 10} = 0.00173\sqrt{x} = 0.00173\sqrt{3} = 0.003$ m.
Assume the streamline is outside the b.l. Continuity is then

$$10\times 0.02 = \int_0^{.003} 10\left(\frac{2y}{.003}-\frac{y^2}{.003^2}\right)dy + (h-.003)10$$

$$= 0.02 + 10h - 0.03. \qquad \therefore h = 0.021 \text{ m or } \underline{2.1 \text{ cm}}$$

$$d_d = \frac{1}{10}\int_0^{.003}\left(10-\frac{20y}{.003}+\frac{10y^2}{.003^2}\right)dy = \frac{1}{10}[.03-.03+.01] = \underline{0.001 \text{ m}}$$

$h - 2 = 2.1 - 2 = 0.1$ cm or $0.001$ m.

The streamline moves away from the wall a distance $d_d$.

8.109 From Prob. 8.107 we found that we should displace the one wall outward $4d_d$.
From the definition of $d_d$:

$$h(x) = 4d_d = \frac{4}{10}\int_0^d \left(10 - \frac{20y}{d} + \frac{10y^2}{d^2}\right)dy = 4\left(d - d + \frac{d}{3}\right) = \frac{4}{3}d$$

$$= \frac{4}{3}\left(5.48\sqrt{\frac{1.86\times 10^{-5}x\ /\ 10}{160\ /\ (.287\times 303)}}\right) = \underline{0.00735\sqrt{x}}\ \text{m}$$

We used $d(x)$ found in Example 8.13, $r = p\ /\ RT$, and $n = m\ /\ r$.

8.110 a) $u = U_\infty\left(\frac{3}{2}\frac{y}{d} - \frac{1}{2}\frac{y^3}{d^3}\right)$. $d_d = \frac{1}{U_\infty}\int_0^d U_\infty\left(1 - \frac{3}{2}\frac{y}{d} + \frac{1}{2}\frac{y^3}{d^3}\right)dy = d - \frac{3}{4}d + \frac{1}{8}d = .375d$.

From Eq. 8.6.16, $d_d = .375\times 4.65\sqrt{\frac{nx}{U_\infty}} = \underline{1.74\sqrt{\frac{nx}{U_\infty}}}$. %error = $\underline{1.2\%}$.

$$q = \frac{1}{U_\infty^2}\int_0^d U_\infty^2\left(\frac{3}{2}\frac{y}{d} - \frac{1}{2}\frac{y^3}{d^3}\right)\left(1 - \frac{3}{2}\frac{y}{d} + \frac{1}{2}\frac{y^3}{d^3}\right)dy = 0.139d.$$

$\therefore q = .139\times 4.65\sqrt{\frac{nx}{U_\infty}} = \underline{0.648\sqrt{\frac{nx}{U_\infty}}}$. %error $= \frac{.648-.644}{.644}\times 100 = \underline{0.62\%}$

b) $u = U_\infty\left(2\frac{y}{d} - \frac{y^2}{d^2}\right)$. See Example 8.13. $d_d = \int_0^d\left(1 - \frac{2y}{d} + \frac{y^2}{d^2}\right)dy = d - d + \frac{d}{3} = d\ /\ 3$.

$\therefore d_d = \frac{5.48}{3}\sqrt{\frac{nx}{U_\infty}} = \underline{1.83\sqrt{\frac{nx}{U_\infty}}}$. %error $= \frac{1.83 - 1.72}{1.72}\times 100 = \underline{6.4\%}$.

$$q = \int_0^d\left(2\frac{y}{d} - \frac{y^2}{d^2}\right)\left(1 - 2\frac{y}{d} + \frac{y^2}{d^2}\right)dy = d - \frac{1}{3}d - \frac{4}{3}d + \frac{2}{4}d + \frac{2}{4}d - \frac{1}{5}d = .1333d.$$

$\therefore q = .1333\times 5.48\sqrt{\frac{nx}{U_\infty}} = \underline{0.731\sqrt{\frac{nx}{U_\infty}}}$. %error $= \frac{.731-.644}{.644}\times 100 = \underline{13.5\%}$.

c) $d_d = \int_0^d\left(1 - \sin\frac{py}{2d}\right)dy = d - \frac{2d}{p} = 0.363d$. See Problem 8.104. $d = 4.79\sqrt{\frac{nx}{U_\infty}}$.

$\therefore d_d = 0.363\times 4.79\sqrt{\frac{nx}{U_\infty}} = \underline{1.74\sqrt{\frac{nx}{U_\infty}}}$. %error $= \frac{1.74 - 1.72}{1.72}\times 100 = \underline{1.2\%}$

$$q = \int_0^d \sin\frac{py}{2d}\left(1 - \sin\frac{py}{2d}\right)dy = \left[-\frac{2d}{p}\cos\frac{py}{2d} - \frac{y}{2} + \text{sin- term}\right]_0^d = -\frac{d}{2} + \frac{2d}{p} = 0.137d.$$

$\therefore q = .137\times 4.79\sqrt{\frac{nx}{U_\infty}} = \underline{0.654\sqrt{\frac{nx}{U_\infty}}}$. %error $= \frac{.654-.644}{.644}\times 100 = \underline{1.6\%}$.

8.111 a) $\delta = 4.65\sqrt{\frac{\nu x}{U_\infty}} = 4.65\left(\frac{1.6\times 10^{-4}\times 20}{12}\right)^{1/2} = \underline{0.0759 \text{ ft}}.$

b) $\tau_0 = .323\rho U_\infty^2\sqrt{\frac{\nu}{xU_\infty}} = .323\times .0024\times 12^2\left(\frac{1.6\times 10^{-4}}{20\times 12}\right)^{1/2} = \underline{9.11\times 10^{-5} \text{ psf}}.$

c) $\text{Drag} = \frac{1}{2}\rho U_\infty^2\times 20\times 15\times 1.29\sqrt{\frac{\nu}{LU_\infty}}$

$$= \frac{1}{2}\times .0024\times 12^2\times 300\times 1.29\left(\frac{1.6\times 10^{-4}}{20\times 12}\right)^{1/2} = \underline{0.0546 \text{ lb}}.$$

d) $\delta_{x=10} = 4.65\sqrt{\frac{1.6\times 10^{-4}\times 10}{12}} = 0.0416 \text{ ft}.\quad \frac{\partial u}{\partial x} = U_\infty\left[-\frac{3y}{2\delta^2}+\frac{3y^3}{2\delta^4}\right]\frac{d\delta}{dx}.$

$$\therefore \frac{\partial u}{\partial x} = 12\left[-\frac{3y}{.2\times .0416^2}+\frac{3y^3}{.2\times .0416^4}\right]\frac{4.65}{2}\sqrt{\frac{1.6\times 10^{-4}}{10\times 12}} = -27.9y + 16140y^3.$$

$$\therefore v = \int_0^\delta -\frac{\partial u}{\partial x}dy = \frac{27.9}{2}\times .0416^2 - \frac{16140}{4}\times .0416^4 = \underline{0.0121 \text{ fps}}.$$

8.112 a) $\delta = 4.65\sqrt{\frac{\nu x}{U_\infty}} = 4.65\left(\frac{1.5\times 10^{-5}\times 6}{4}\right)^{1/2} = \underline{0.0221 \text{ m}}.$

b) $\tau_0 = 0.323\rho U_\infty^2\sqrt{\frac{\nu}{xU_\infty}} = .323\times 1.22\times 4^2\left(\frac{1.5\times 10^{-5}}{6\times 4}\right)^{1/2} = \underline{0.00498 \text{ Pa}}.$

c) $\text{Drag} = \frac{1}{2}\rho U_\infty^2 Lw\times 1.29\sqrt{\frac{\nu}{LU_\infty}} = \frac{1}{2}\times 1.22\times 4^2\times 6\times 5\times 1.29\left(\frac{1.5\times 10^{-5}}{6\times 4}\right)^{1/2} = \underline{0.299 \text{ N}}.$

d) $\frac{\partial u}{\partial x} = U_\infty\left[-\frac{3y}{2\delta^2}+\frac{3}{2}\frac{y^3}{\delta^4}\right]\frac{d\delta}{dx}$

$$= 4\left[-\frac{3\times 4}{2\times 4.65^2\times 1.5\times 10^{-5}\times 3}y + \frac{3}{2}\frac{4^2y^3}{4.65^4\times(1.5\times 10^{-5}\times 3)^2}\right]\frac{d\delta}{dx}.$$

$$\therefore \frac{\partial u}{\partial x} = 4\left[-6166y + 2.53\times 10^7y^3\right]\frac{4.65}{2}\sqrt{\frac{1.5\times 10^{-5}}{4}}\frac{1}{\sqrt{3}} = -64.1y + 2.63\times 10^5y^3.$$

$$\therefore v = \int_0^\delta -\frac{\partial u}{\partial x}dy = \frac{64.1}{2}\times .0156^2 - \frac{2.63\times 10^5}{4}\times .0156^4 = \underline{0.00391 \text{ m/s}},$$

where $\delta_{x=3} = 4.65\sqrt{\frac{1.5\times 10^{-5}\times 3}{4}} = .01560 \text{ m}.$

8.113 a) $\delta = 5\sqrt{\frac{\nu x}{U_\infty}} = 5\left(\frac{1.5\times10^{-5}\times2}{10}\right)^{1/2} = \underline{0.00866\text{ m}}$. Use $\tau_0 = .332\rho U_\infty^2\sqrt{\frac{\nu}{xU_\infty}}$.

$$\text{Drag} = \int_0^L \tau_0 w dx = .332\times1.22\times10^2\sqrt{\frac{1.5\times10^{-5}}{10}}\frac{\sqrt{2}}{1/2}\times4 = \underline{0.561\text{ N}}.$$

b) $\delta = .38\times2\left(\frac{1.5\times10^{-5}}{10\times2}\right)^{.2} = \underline{0.0453\text{ m}}$.

$$\text{Drag} = \frac{1}{2}\rho U_\infty^2 Lw\times.074\left(\frac{\nu}{U_\infty L}\right)^{.2} = \frac{1}{2}\times1.22\times10^2\times2\times4\times.074\left(\frac{1.5\times10^{-5}}{10\times2}\right)^{.2} = \underline{2.15\text{ N}}.$$

8.114 a) $\delta = .38\times6\left(\frac{1.5\times10^{-5}}{20\times6}\right)^{.2} = \underline{0.0949\text{ m}}$. $\tau_0 = \frac{1}{2}\times1.22\times20^2\times.059\left(\frac{1.5\times10^{-5}}{20\times6}\right)^{.2}$

$= \underline{.6\text{ Pa}}$.

b) $\delta = .38\times6\left(\frac{10^{-6}}{20\times6}\right)^{.2} = \underline{0.0552\text{ m}}$. $\tau_0 = \frac{1}{2}\times1000\times20^2\times.059\left(\frac{10^{-6}}{20\times6}\right)^{.2}$

$= \underline{286\text{ Pa}}$.

8.115 $u(y=\delta) = U_\infty$. $\frac{\partial u}{\partial y} = \frac{1}{7}U_\infty y^{-6/7}\delta^{-1/7}$. $\left.\frac{\partial u}{\partial y}\right|_{y=\delta} = \frac{1}{7}U_\infty/\delta$.

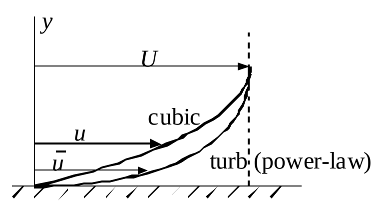
$\left.\frac{\partial u}{\partial y}\right|_{y=\delta}$ should be zero. Thus, this condition is not satisfied.

$\tau_0 = \mu\left.\frac{\partial u}{\partial y}\right|_{y=0} = \mu\frac{1}{7}U_\infty\frac{1}{0}\delta^{-1/7} = \infty$. Thus, this is unacceptable and

$\frac{\partial u}{\partial y}$ at, and near, the wall is not valid.

$$u = U_\infty\left(\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\frac{y^3}{\delta^3}\right).$$

$$\bar{u} = U_\infty\left(\frac{y}{\delta}\right)^{1/7}.$$




8.116 a) Drag =

$$\frac{1}{2}\times.0024\times20^2\times(12\times15)\left[.074\left(\frac{1.58\times10^{-4}}{20\times12}\right)^{.2} - 1060\left(\frac{1.58\times10^{-4}}{20\times12}\right)\right] = \underline{0.31\text{ lb}}.$$

b) Drag =

$$\frac{1}{2}\times .0024\times 20^2\times(12\times 15)\left[.074\left(\frac{1.58\times 10^{-4}}{20\times 12}\right)^{.2}-1700\left(\frac{1.58\times 10^{-4}}{20\times 12}\right)\right]=\underline{0.27\text{ lb}}.$$

c) Drag =

$$\frac{1}{2}\times .0024\times 20^2\times(12\times 15)\left[.074\left(\frac{1.58\times 10^{-4}}{20\times 12}\right)^{.2}-2080\left(\frac{1.58\times 10^{-4}}{20\times 12}\right)\right]=\underline{0.25\text{ lb}}.$$

8.117 a) Drag = $\frac{1}{2}\times 1000\times 1.2^2\times(1\times 2)\left[.074\left(\frac{10^{-6}}{1.2\times 1}\right)^{.2}-1060\left(\frac{10^{-6}}{1.2\times 1}\right)\right]=\underline{5.21\text{ N}}.$

b) Drag = $\frac{1}{2}\times 1000\times 1.2^2\times(1\times 2)\left[.074\left(\frac{10^{-6}}{1.2\times 1}\right)^{.2}-1700\left(\frac{10^{-6}}{1.2\times 1}\right)\right]=\underline{4.44\text{ N}}.$

c) Drag = $\frac{1}{2}\times 1000\times 1.2^2\times(1\times 2)\left[.074\left(\frac{10^{-6}}{1.2\times 1}\right)^{.2}-2080\left(\frac{10^{-6}}{1.2\times 1}\right)\right]=\underline{3.99\text{ N}}.$

8.118 $U_\infty = 60\frac{1000}{3600}=16.67\text{ m / s.}\quad d=.38\times 100\,000\left(\frac{1.5\times 10^{-5}}{16.67\times 10^5}\right)^{.2}=\underline{235\text{ m}}.$

$$t_0=\frac{1}{2}rU_\infty^2c_f=\frac{1}{2}\times 1.22\times 16.67^2\times\left[.059\left(\frac{1.5\times 10^{-5}}{16.67\times 10^5}\right)^{.2}\right]=\underline{0.0618\text{ Pa}}.$$

b) $t_0=\frac{1}{2}rU_\infty^2c_f=\frac{1}{2}\times 1.22\times 16.67^2\frac{.455}{\left[\ell n\left(.06\frac{16.67\times 10^5}{1.5\times 10^{-5}}\right)\right]^2}=\underline{0.151\text{ Pa}}.$

$$\therefore u_t=\sqrt{\frac{.151}{1.22}}=.351\text{ m / s.}\quad\therefore\frac{16.67}{.351}=2.44\ell n\frac{.351d}{1.5\times 10^{-5}}+7.4.\quad\therefore \underline{d=585\text{ m}}.$$

Both (a) and (b) are in error, however, (b) is more accurate. $\frac{¶ p}{¶ x}<0.$

8.119 a) $5=\frac{u_t d_n}{n}$ (See Fig. 8.24 b). $\therefore d_n=\frac{5\times 1.5\times 10^{-5}}{.351}=\underline{2.14\times 10^{-4}\text{ m}}.$

b) $d_d=\frac{1}{U_\infty}\int_0^d(U_\infty-\bar{u})dy=\frac{m_t}{U_\infty}\int_{d_n}^{.15d}\left(2.5-2.44\ell n\frac{y}{d}\right)dy+\frac{u_t}{U_\infty}\int_{.15d}^{d}-3.74\ell n\frac{y}{d}dy$

$$=\frac{u_t}{U_\infty}\left[2.5(.15d-d_n)-2.44\left(y\ell n\frac{y}{d}-y\right)\Bigg|_{d_v}^{.15d=87.8}-3.74\left(y\ell n\frac{y}{d}-y\right)\Bigg|_{87.8}^{d=585}\right]$$

$$=\frac{.351}{16.67}[219+620-.008+2188-951]=\underline{43.7\text{ m}}.$$

Note: We cannot use zero as a lower limit since the ln-profile does not go to the wall. Hence, we use $d_n$; the lower limit provides a negligible contribution to the integral.

8.120 a) Use Eq. 8.6.40: $c_f = \dfrac{.455}{\left[\ell n\left(.06\dfrac{300\times 20}{1.58\times 10^{-4}}\right)\right]^2} = \underline{0.00212}.$

b) $t_0 = \frac{1}{2} rU_\infty^2 c_f = \frac{1}{2}\times .0024\times 300^2 \times .00212 = \underline{0.229 \text{ psf}}.$ $\quad u_t = \sqrt{\dfrac{.229}{.0024}} = 9.77 \text{ fps}.$

c) $d_n = \dfrac{5n}{u_t} = 5\times 1.58\times 10^{-4} \;/\; 9.77 = \underline{8.09\times 10^{-5} \text{ ft}}.$

d) $\dfrac{300}{9.77} = 2.44\ell n\dfrac{9.77d}{1.58\times 10^{-4}} + 7.4. \qquad \therefore \underline{d = 0.228 \text{ ft}}.$

8.121 a) $t_0 = \frac{1}{2} rU_\infty^2 c_f = \frac{1}{2}\times 1000\times 10^2 \dfrac{.455}{\left[\ell n\left(.06\dfrac{10\times 3}{10^{-6}}\right)\right]^2} = 110 \text{ Pa}.$

$\therefore u_t = \sqrt{\dfrac{110}{1000}} = .332 \text{ m / s}. \qquad \therefore d_n = \dfrac{5n}{u_t} = \dfrac{5\times 10^{-6}}{.332} = \underline{1.51\times 10^{-5} \text{ m}}.$

b) $\bar{u} = 5u_t = 5\times .332 = \underline{1.66 \text{ m / s}}.$

c) $y = .15d.$ — Do part (d) first! $\qquad \therefore y = .15\times .0333 = \underline{0.005 \text{ m}}.$

d) $\dfrac{10}{.332} = 2.44\ell n\dfrac{.332d}{10^{-6}} + 7.4. \qquad \therefore \underline{d = 0.0333 \text{ m}}.$

8.122 Assume flat plates with $dp/dx = 0$. $\quad C_f = \dfrac{.523}{\left[\ell n\left(.06\dfrac{10\times 100}{10^{-6}}\right)\right]^2} = .00163.$

$\therefore \text{Drag} = 2\times\frac{1}{2}\times 1000\times 10^2 \times 10\times 100\times .00163 = \underline{163\,000 \text{ N}}.$

To find $d_{max}$ we need $u_t$.

$t_0 = \frac{1}{2}\times 1000\times 10^2 \dfrac{.455}{\left[\ell n\left(.06\dfrac{10\times 100}{10^{-6}}\right)\right]^2} = 70.9 \text{ Pa}. \quad \therefore u_t = \sqrt{\dfrac{70.9}{1000}} = 0.266 \text{ m / s}.$

$$\frac{10}{.266} = 2.44\ell n\frac{.266d}{10^{-6}} + 7.4. \qquad \therefore \underline{d_{max} = 0.89 \text{ m}}.$$

8.123 a) Assume a flat plate of width $pD$. $\quad \text{Re} = \dfrac{UL}{n} = \dfrac{15\times 600}{1.5\times 10^{-5}} = 6\times 10^8.$

$$\text{drag} = C_f \frac{1}{2}\rho U^2 L\pi D = 0.073(6\times10^8)^{-1/5}\times\frac{1}{2}\times1.2\times15^2\times600\times\pi\times100 = 32600 \text{ N}$$

$$\text{power} = F_D \times U = 32600\times15 = 489000 \text{ W or } 655 \text{ hp or } \underline{164 \text{ hp/engine}}.$$

b) $\rho_{\text{helium}} = \dfrac{p}{RT} = \dfrac{100}{2.077\times288} = 0.167 \text{ kg/m}^3$.

$$F_B = W_{\text{air}} - W_{\text{helium}} = \Delta\rho\times\mathcal{V} = (1.2-0.167)\times9.8\times\pi\times50^2\times600/2 = 2.38\times10^7$$

$$\text{payload} = F_B - W = 23.8\times10^6 - 9.8\times1.2\times10^6 = \underline{12\times10^6 \text{ N}}$$

8.124 $u = \dfrac{\partial\psi}{\partial y}, \quad \dfrac{\partial u}{\partial x} = \dfrac{\partial^2\psi}{\partial x\partial y}, \quad v = -\dfrac{\partial\psi}{\partial x}, \quad \dfrac{\partial u}{\partial y} = \dfrac{\partial^2\psi}{\partial y^2}, \quad \dfrac{\partial^2 u}{\partial y^2} = \dfrac{\partial^3\psi}{\partial y^3}.$

Substitute into Eq. 8.6.45 (with $dp/dx = 0$):

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} = \nu\frac{\partial^3\psi}{\partial y^3}.$$

8.125 We also have

$$\frac{\partial\psi}{\partial x} = \frac{\partial\psi}{\partial\xi}\frac{\partial\xi}{\partial x} + \frac{\partial\psi}{\partial\eta}\frac{\partial\eta}{\partial x}, \quad \frac{\partial^2\psi}{\partial x\partial y} = \frac{\partial(\partial\psi/\partial y)}{\partial\xi}\frac{\partial\xi}{\partial x} + \frac{\partial(\partial\psi/\partial y)}{\partial\eta}\frac{\partial\eta}{\partial x}$$

Recognizing that $\partial\xi/\partial x = 1$, $\partial\xi/\partial y = 0$, $\partial\eta/\partial x = -\dfrac{y}{2}\sqrt{U_\infty/\nu x^3}$, and

$$\partial\eta/\partial y = \sqrt{U_\infty/\nu x}, \qquad \frac{\partial\psi}{\partial y} = \sqrt{\frac{U_\infty}{\nu\xi}}\frac{\partial\psi}{\partial\eta}, \quad \frac{\partial\psi}{\partial x} = \frac{\partial\psi}{\partial\xi} - \frac{y}{2}\sqrt{\frac{U_\infty}{\nu x^3}}\frac{\partial\psi}{\partial\eta}$$

$$\frac{\partial^2\psi}{\partial x\partial y} = \sqrt{\frac{U_\infty}{\nu\xi}}\frac{\partial^2\psi}{\partial\xi\partial\eta} - \frac{1}{2}\sqrt{\frac{U_\infty}{\nu\xi^3}}\frac{\partial\psi}{\partial\eta} + \sqrt{\frac{U_\infty}{\nu\xi}}\frac{\partial^2\psi}{\partial\eta^2}\left(-\frac{y}{2}\sqrt{\frac{U_\infty}{\nu\xi^3}}\right)$$

$$\frac{\partial^2\psi}{\partial y^2} = \sqrt{\frac{U_\infty}{\nu\xi}}\frac{\partial^2\psi}{\partial\eta^2}\left(\sqrt{\frac{U_\infty}{\nu\xi}}\right), \qquad \frac{\partial^3\psi}{\partial y^3} = \frac{U_\infty}{\nu\xi}\frac{\partial^3\psi}{\partial\eta^3}\sqrt{\frac{U_\infty}{\nu\xi}}$$

Equation 8.6.47 then becomes, using $\sqrt{U_\infty/\nu\xi} = \eta/y$,

$$\frac{\eta}{y}\frac{\partial\psi}{\partial\eta}\left(\frac{\eta}{y}\frac{\partial^2\psi}{\partial\xi\partial\eta} - \frac{\eta}{2yx}\frac{\partial\psi}{\partial\eta} - \frac{\eta^2}{2yx}\frac{\partial^2\psi}{\partial\eta^2}\right) - \left(\frac{\partial\psi}{\partial\xi} - \frac{\eta}{2x}\frac{\partial\psi}{\partial\eta}\right)\left(\frac{\eta^2}{y^2}\right)\frac{\partial^2\psi}{\partial\eta^2}$$

$$= \nu\frac{U_\infty}{\nu x}\frac{\eta}{y}\frac{\partial^3\psi}{\partial\eta^3}$$

Multiply by $y^2/\eta^2$ and Eq. 8.6.49 results:

$$-\frac{1}{2\xi}\left(\frac{\partial\psi}{\partial\eta}\right)^2 + \frac{\partial^2\psi}{\partial\xi\partial\eta}\frac{\partial\psi}{\partial\eta} - \frac{\partial\psi}{\partial\xi}\frac{\partial^2\psi}{\partial\eta^2} = \nu\frac{\partial^3\psi}{\partial\eta^3}\sqrt{\frac{U_\infty}{\nu\xi}}$$

8.126 $u = \frac{\partial \psi}{\partial y} = \sqrt{U_\infty \nu x}\,\frac{dF}{d\eta}\frac{\partial \eta}{\partial y} = \sqrt{U_\infty \nu x}\; F'(\eta)\sqrt{\frac{U_\infty}{\nu x}} = \underline{U_\infty F'(\eta)}.$

We used Eq. 8.6.50 and Eqs. 8.6.48.

$$v = -\frac{\partial \psi}{\partial x} = -\frac{\partial}{\partial x}\left(\sqrt{U_\infty \nu x}\; F\right) = -\frac{1}{2}\sqrt{\frac{U_\infty \nu}{x}}\,F - \sqrt{U_\infty \nu x}\;\frac{\partial F}{\partial \eta}\frac{\partial \eta}{\partial x}$$

$$= -\frac{1}{2}\sqrt{\frac{U_\infty \nu}{x}}\,F - \sqrt{U_\infty \nu x}\,F'\; y\sqrt{\frac{U_\infty}{\nu}}\left(-\frac{1}{2}x^{-3/2}\right)$$

$$= -\frac{1}{2}\sqrt{\frac{U_\infty \nu}{x}}\,F + \frac{y}{2}\sqrt{\frac{U_\infty}{\nu x}}\sqrt{\frac{U_\infty \nu}{x}}\;F' = \underline{\frac{1}{2}\sqrt{\frac{U_\infty \nu}{x}}(\eta F' - F)}.$$

8.127 The results are shown in Table 8.5.

8.128 a) $\tau_0 = 0.332 \times 1.22 \times 5^2\sqrt{\frac{1.5\times 10^{-5}}{2\times 5}} = \underline{0.0124\ \text{Pa}}.$

b) $\delta = 5\sqrt{\frac{1.5\times 10^{-5}\times 2}{5}} = \underline{0.0122\ \text{m}}.$

c) $v_{max} = \sqrt{\frac{\nu U_\infty}{x}}\left[\frac{1}{2}(\eta F' - F)\right]_{max} = \sqrt{\frac{1.5\times 10^{-5}\times 5}{2}}\times .8605 = \underline{0.00527\ \text{m/s}}.$

d) $Q = \int_0^\delta u\,dy = \int_0^\delta U_\infty \frac{dF}{d\eta}dy = \int_0^\delta U_\infty \frac{dF}{d\eta}d\eta\sqrt{\frac{\nu x}{U_\infty}}$

$$= U_\infty\sqrt{\frac{\nu x}{U_\infty}}[F(\delta) - F(0)] = 5\sqrt{\frac{1.5\times 10^{-5}\times 2}{5}}\times 3.28 = \underline{0.04\ \text{m}^2/\text{s}/\text{m}}$$

8.129 a) $\tau_0 = .332 \times .0024 \times 15^2\sqrt{\frac{1.6\times 10^{-4}}{6\times 15}} = \underline{2.39\times 10^{-4}\ \text{psf}}.$

b) $\delta = 5\sqrt{\frac{1.6\times 10^{-4}\times 6}{15}} = 0.04\ \text{ft}.$

c) $v_{max} = \sqrt{\frac{\nu U_\infty}{x}}\left[\frac{1}{2}(\eta F' - F)\right]_{max} = \sqrt{\frac{1.6\times 10^{-4}\times 15}{6}}\times .8605 = \underline{0.0172\ \text{fps}}.$

d) $Q = \int_0^\delta u\,dy = U_\infty\sqrt{\frac{\nu x}{U_\infty}}F(\delta) = 15\sqrt{\frac{1.6\times 10^{-4}\times 6}{15}}\times 3.28 = \underline{0.394\ \text{ft}^2/\text{sec}/\text{ft}}.$

8.130 At $x$ = 2m, Re = $5 \times 2/10^{-6} = 10^7$. $\therefore$ Assume turbulent from the leading edge.

a) $\tau_0 = \frac{1}{2}\rho U_\infty^2 \frac{0.455}{(\ell n\ 0.06\,\text{Re}_x)^2}$

$$= \frac{1}{2} \times 1000 \times 5^2 \, \frac{0.455}{(\ell n \; 0.06 \times 10^7)^2} = \underline{32.1 \text{ Pa}}$$

b) $u_\tau = \sqrt{\tau_0 / \rho} = \sqrt{32.1 / 1000} = 0.1792 \text{ m / s}$

$$\frac{5}{0.1792} = 2.44 \ell n \, \frac{0.1792\delta}{10^{-6}} + 7.4. \qquad \therefore \delta = 0.0248 \text{ m or } \underline{24.8 \text{ mm}}$$

c) Use the 1/7 the power-law equation:

$$Q = \int_0^{0.0248} 5(y / 0.0248)^{1/7} dy = \underline{0.109 \text{ m}^3 \text{ / s / m}}$$

8.131 From Table 8.5 we would select $\eta = 6$:

a) $\delta = 6\sqrt{\frac{\nu x}{U_\infty}} = 6\sqrt{\frac{1.5 \times 10^{-5} \times 2}{5}} = \underline{0.0147 \text{ m}}$

b) $\delta = 6\sqrt{\frac{\nu x}{U_\infty}} = 6\sqrt{\frac{15.8 \times 10^{-5} \times 6}{15}} = \underline{0.047 \text{ ft}}$ or $\underline{0.57 \text{ in.}}$

8.132 From Table 8.5 we interpolate for $F' = 0.5$ to be

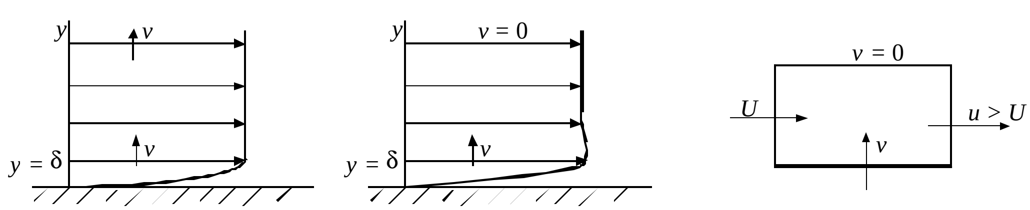
$$\eta = \frac{0.5 - 0.3298}{0.6298 - 0.3298}(2 - 1) + 1 = 1.57$$

$$= y\sqrt{\frac{5}{1.5 \times 10^{-5} \times 2}}. \qquad \therefore y = 0.00385 \text{ m or } \underline{3.85 \text{ mm}}$$

$$v = \sqrt{\frac{\nu U_\infty}{x}}\left(\frac{1}{2}\right)(\eta F' - F) \quad = \sqrt{\frac{1.5 \times 10^{-5} \times 5}{2}}(0.207) = \underline{0.00127 \text{ m / s}}$$

$$\tau = \mu\left(\frac{\partial u}{\partial y} + \cancel{\frac{\partial v}{\partial x}}\right) = F''\rho U_\infty^2\sqrt{\frac{\nu}{xU_\infty}} = 0.291(1.2)5^2\sqrt{\frac{1.5 \times 10^{-5}}{2 \times 5}} = \underline{0.011 \text{ Pa}}$$

8.133

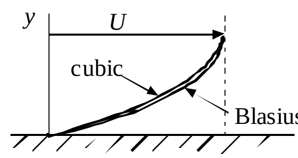


(a) (b) (c)

If $v = 0$ at $y = 10\delta$ and $v > 0$ at $y = \delta$, then $\partial v / \partial y < 0$ and continuity demands that $\partial u / \partial x > 0$. The $u$ component, for $y > \delta$ must then be greater than $U$, as shown in (b); there should be a slight "overshoot". Also, consider the control volume of (c) where the lower boundary is just above $y = \delta$. If $v = 0$ at large $y$, say $y = 10\delta$, then continuity demands that $u$ out the right area be greater than $U$: an "overshoot". It is not reasonable to assume that $v$ = const as in (a);

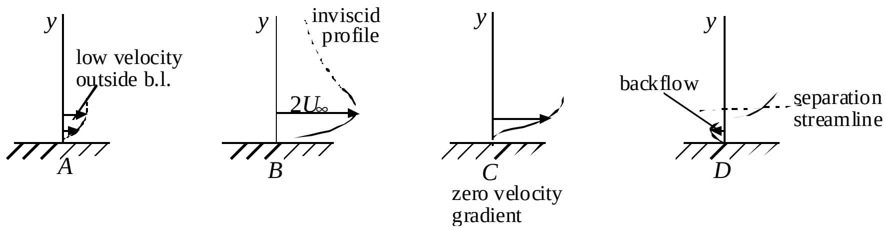
reality would demand a profile such as that sketched in (b). The overshoot would be quite small and is neglected in boundary layer theory.

8.134 $u = U_{\infty}\left(\frac{3}{2}\frac{y}{d} - \frac{1}{2}\frac{y^3}{d^3}\right)$




For the Blasius profile: see Table 8.5.
(This is only a sketch. The student is encouraged to draw the profiles to scale.)

8.135




8.136 A: $\frac{\P p}{\P x} < 0.$ (favorable)

B: $\frac{\P p}{\P x} \cong 0.$

C: $\frac{\P p}{\P x} > 0.$ (unfavorable)

D: $\frac{\P p}{\P x} > 0.$

E: $\frac{\P p}{\P x} < 0.$